About Cluster Problems

Cluster problems are a set of problems that encourage students to think about what they know in order to use that knowledge to solve harder problems. The cluster problems in this unit are designed to help students become familiar with different strategies for multiplying and dividing. Through these cluster problems, students learn how they can pull apart problems into more manageable components. They learn to use multiplication combinations they know and the relationship between multiplication and division to solve problems. They focus on understanding multiplying and dividing by multiples of 10 and 100, a critical skill in solving multidigit problems.

Different clusters suggest different strategies. For example, here is a cluster of problems that includes 2 different ways to think about solving 59 × 92.

$50 \times 90 =$

$60 \times 90 =$

$9 \times 90 =$

$59 \times 90 =$

$59 \times 2 =$

The problems in this cluster include the possibility of breaking 59 × 92 apart to create partial products (50 × 90, 9 × 90, 59 × 2), or to make an easier problem by changing 59 to 60 (60 × 90) and then compensating by subtracting one group of 92. Cluster problems often include more than one strategy for students to consider.

Students solve all of the problems in the cluster and then think about what strategy makes the most sense and is the most efficient to solve the final problem.

Cluster problems are intended to help students learn how to look at a problem and build a strategy to solve it that is based on the number relationships they know. Students work on clusters of problems that are provided for them. They also create their own clusters of multiplication problems. Throughout their work on cluster problems in this unit, encourage students to add to the clusters any other problems they use to solve the final problem in the cluster. (Students will encounter division cluster problems in Grade 5.)

As students solve multiplication problems in this unit, observe their strategies and ways of thinking about multiplication. Encourage them to use what they already know to create their own clusters and to find solutions. Ask them to explain what parts of the problem they have solved and what parts are still remaining. As they build on what they already understand well (such as the basic multiplication combinations, multiplying by multiples of 10, and other multiplication problems that they can solve easily), they deepen their understanding of multiplication and become increasingly efficient and flexible in solving problems.

Multiplication Clusters and the Properties of Multiplication

Multiplication clusters are sets of problems that help students think about using what they know to solve harder problems. For example, what do you know that helps you solve 12 × 3? If you know that 3 × 3 = 9, you can double the product of 3 × 3 to get the product of 6 × 3 and then double again to get the product of 12 × 3. You might also start with 10 × 3. If you know that 10 × 3 = 30, then you can start with 30 and add two more 3s to get 36. As students work with multiplication clusters, they learn to think about all the number relationships they know that might help them solve a problem.

The multiplication clusters in this unit are designed to help students make sense of multiplying 2-digit numbers. Many of the clusters build an understanding of pulling apart multiplication problems into manageable subproblems, solving the smaller problems, and then putting the parts back together. This process is based on an important characteristic of multiplication called the *distributive property.* In this unit, students are not expected to learn the name of the property, but it is a core idea of the unit.

Here is an example:

$$6 \times 23 = (6 \times 10) + (6 \times 10) + (6 \times 3)$$

In this example, 23 is broken apart into 10 + 10 + 3, and *each part* is multiplied by 6 in order to construct the solution to 6 × 23. The number does not have to be split into 10s and 1s.

Here is another example:

$$8 \times 12 = (4 \times 12) + (4 \times 12)$$

or

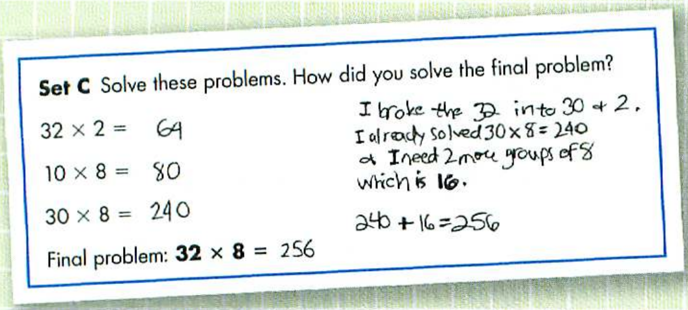
$$8 \times 12 = (8 \times 6) + (8 \times 6)$$

In each case, one of the factors is split up into parts, and each part is multiplied by the other factor in order to maintain equivalence to the original expression.

Other clusters build on ideas about halving and doubling that are developed in this unit. See **Teacher Note:** Reasoning and Proof in Mathematics?, page 168, for more about students' understanding of creating an equivalent multiplication problem by halving one factor and doubling the other.

As students solve the first few problems in each cluster, they use familiar multiplication combinations. Students say "I just knew it" for some of the problems because these single-digit multiplication combinations are part of their known repertoire. They also make use of multiplying by 10 and by multiples of 10, another essential tool in solving harder multiplication problems. See **Teacher Note:** Multiplying by Multiples of 10, page 167, for more about the ways students develop understanding of this idea.

Here are examples of student work on two multiplication clusters from the *Student Activity Book* pages 57–58.


Set C Solve these problems. How did you solve the final problem?

32 × 2 = 64

10 × 8 = 80

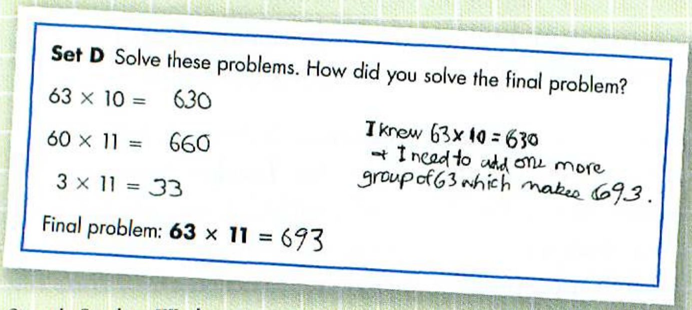
30 × 8 = 240

Final problem: **32** × **8** = 256

I broke the 32 into 30 + 2. I already solved 30 × 8 = 240 + I need 2 more groups of 8 which is 16.

240 + 16 = 256

Sample Student Work


Set D Solve these problems. How did you solve the final problem?

63 × 10 = 630

60 × 11 = 660

3 × 11 = 33

Final problem: **63** × **11** = 693

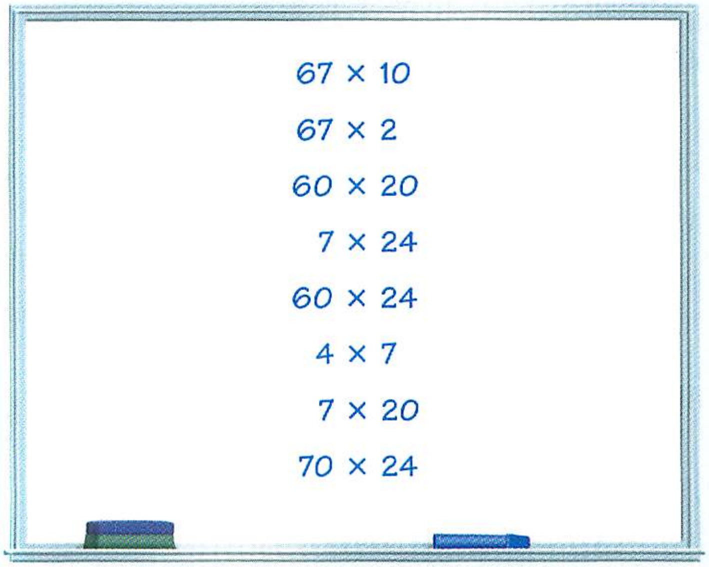
I knew 63 × 10 = 630 + I need to add one more group of 63 which makes 693.

Sample Student Work

Multiplication clusters help students learn how to look at a problem and build a strategy to solve it that is based on the number relationships they know. At first, students work on clusters of problems that are provided to help them solve a 2-digit problem, such as 4 × 43 or 58 × 6. They solve all the problems in the cluster and then decide which one(s) will most help them think about the solution to the final problem. Students may add to the cluster any other problems that help them solve the final one. Later in the unit, students create their own cluster of problems to help them solve a multiplication problem. In later units of *Investigations,* in both Grades 4 and 5, students spend more time creating their own clusters of problems as well as using a variety of given problems to solve multiplication and division problems.

Creating a Cluster Problem

Students have been working on using clusters of related problems to help them solve 2-digit by 2-digit multiplication problems. They are now making cluster problems of their own for the problem 67×24. The teacher has collected the following set of problems related to 67×24 from various students:



Teacher: All of these problems can help you solve 67 times 24. Who would like to tell us which ones you used?

Terrell: I used 70×24. Then, from the answer I got, I took off 3×24.

Teacher: How did you find the answer to 70×24?

Terrell: I really thought of it as 4×25 and then 3×25, only it's 40×25 and 30×25. That's 1,000 plus 750, so take away 70 because it's 24, not 25, and that's 1,680. Then I subtracted 3×24.

Teacher: So let's add 70×25 to our list because that helped you too.

Ramona: I did 67×10 two times and then 67×2 two times. 670, 670, 134, 134—add them all up and it's 1,608.

Luke: I used 4 problems: $60 \times 20 = 1{,}200$; $7 \times 4 = 28$; $60 \times 4 = 240$; and $7 \times 20 = 140$. The only one that's not up there is 60×4. (The teacher adds 60×4 to the list.)

Teacher: Let's look at Luke's solution. Where did the 60 in 60×20 and 60×4 come from?

Jill: From 67. He broke it up into 60 and 7.

Teacher: Then what did he do with those 2 parts of the number?

Enrique: He had to multiply them times both parts of 24—that's 20 and 4. So he ended up with 4 problems to solve.

Teacher: Why does it make sense that there are 4 problems, doing it Luke's way?

Bill: It's like if you had 67 teams with 24 on each team, and first you had to put 20 kids on 60 teams, and then 20 kids on the other 7 teams . . .

Anna: But then you still had to put 4 more kids on each team. Four kids on 60 teams, and four kids on 7 teams. So all the teams have 24 kids!

As the students continue to share their solutions, the teacher points out how different problems in the cluster lead to a variety of ways to solve 67×24. This group of students has made use of the following strategies: making an easier problem and compensating for the change [Terrell] and breaking the problem apart into smaller, more manageable problems while keeping track of all the parts of the problem [Ramona and Luke].

Solving Related Multiplication Problems

Here are two related problems.

Cats have 4 legs.
How many legs are on 5 cats?
How many legs are on 8 cats?

Ines solved both problems. She used the answer to the first problem to help her figure out the answer to the second problem.

How many legs are on 5 cats?

5 cats
|||| |||| |||| |||| ||||
20 legs
$5 \times 4 = 20$

How many legs are on 8 cats?

5 cats
|||| |||| |||| |||| ||||
20 legs
$5 \times 4 = 20$

3 cats
|||| |||| ||||
12 legs
$3 \times 4 = 12$

5 cats + 3 cats = 8 cats
|||| |||| |||| |||| |||| + |||| |||| ||||
20 legs + 12 legs = 32 legs
$8 \times 4 = 32$

How would you solve these related problems?
Spiders have 8 legs.
- **How many legs are on 3 spiders?**
- **How many legs are on 6 spiders?**

Multiplication and Division Cluster Problems

Cluster problems help you use what you know about easier problems to solve harder problems.

1. Solve the problems in each cluster.
2. Use one or more of the problems in the cluster to solve the final problem, along with other problems if you need them.

Solve these cluster problems:	How did you solve the final problem?
$24 \times 10 = \underline{240}$ $24 \times 3 = \underline{72}$ $24 \times 20 = \underline{480}$ $24 \times 30 = \underline{720}$ Now solve this problem: $24 \times 31 = \underline{744}$	*I figured out that 24 × 30 would be 720 because 24 × 10 = 240, and 240 + 240 + 240 = 720.* *I need one more group of 24. That's 720 + 24 = 744.* *So, 24 × 31 = 744.*

Solve these cluster problems:	How did you solve the final problem?
$10 \times 12 = \underline{120}$ $5 \times 12 = \underline{60}$ Now solve this problem: $192 \div 12 = \underline{16}$	*I thought of 192 ÷ 12 as ____ × 12 = 192. 10 × 12 = 120 and 5 × 12 = 60, so 15 × 12 = 120 + 60 = 180.* *I need one more 12 to get to 192. 16 × 12 = 192* *So, 192 ÷ 12 = 16.*

Solve these cluster problems:	How did you solve the final problem?
$54 \div 6 = \underline{9}$ $540 \div 6 = \underline{90}$ Now solve this problem: $6\overline{)546}$ = **91**	*After I knew 540 ÷ 6 = 90, then I knew I needed one more group of 6 because 546 = 540 + 6.* *So, 546 ÷ 6 = 91.*