The Case of Ellen: Deciding When to Nudge

Katrina Sajak, like many teachers, strives to help move students forward with their mathematical thinking but often grapples with when to explicitly show students a particular strategy and when to let them work at their own pace. In this case, Ms. Sajak reflects on the decision she made to help Ellen develop a more efficient strategy for solving division story problems.

Every year as I get to know the students in my class, I typically face the decision about when to nudge them and when to let them work at the place they are. As I observe the way they learn and the way they approach mathematics, along with the skills and attitudes they bring, I also notice their confidence and tolerance for challenge. Some students are rather courageous and love as much challenge as possible. Others are reluctant, and some of those students bring with them the anxiety and insecurities of past experiences. Ellen was one such student.

Because I knew she needed a gentle touch, I began the year by treading lightly. After watching her for several weeks, I began to develop insights about Ellen as a learner. She rarely volunteered during math discussions, preferred not to be noticed during math work times, and claimed she did not like math and was terrible at doing it. I could see she had some glitches in her understanding of the number system.

She wasn't yet reliably using tens, and she had difficulty crossing the threshold of 100. Once in the 100s, though, she could manage pretty well until 200. Yet, Ellen also had some obvious strengths. It was hard for me to completely know what kind of support would help her most; but I knew whatever it was, it would have to start from a place of trust and understanding.

By November, things were moving along pretty well for Ellen in mathematics. She trusted herself more and also trusted that my confidence in her as a math student was genuine. She was gaining confidence and had a sense of herself as being competent to do math. Although very hardworking and industrious, she could still be a bit reluctant when she sensed that the ideas were advancing faster than she wanted. Despite her protests, she continued keeping pace and making progress. She was even surprising herself. It was in this mode that I knew I could again challenge her comfort level, as I had at other points during the first semester.

My students had authored division story problems for the class multiplication and division book and were solving each other's problems. I watched carefully as Ellen began. When the numbers were in a range that were workable or the numbers were "friendly" (such as $15 \div 5$ or $6 \div 2$), she was able to work confidently and with more efficiency. However, as soon as the numbers got slightly bigger or less familiar to her (splitting 18 muffins equally among 3 friends), she went back to making unbundled tally marks.

Earlier in the year, tally marks were her preferred strategy. In addition and subtraction she had moved on to greater efficiency. The number line work we did with those operations helped her make the leap to using multiples of 10. Now, however, with these less familiar operations, she was again less certain. For some students who need to make tally marks, I might have tried some other intervention to move them to greater efficiency; but with Ellen I felt confident that she knew more than she was accessing. I approached her desk with a plan. She was working on the following problem:

Ken has 52 toy trucks. He wants to invite 3 friends to his house to play with him and the trucks. For Ken and his friends to each get an equal number of trucks, how many trucks should each child get?

Ellen had written the equation 52 ÷ ___ = 4 for the problem and had drawn all 52 tally marks. She was beginning to circle them in groups of 12, starting from right to left. When I came over to her desk, she had already started erasing some of the circles, indicating that she knew there would be some extras left over after she circled the last 12. She told me she just guessed at 12 but that wouldn't work. Ellen had developed such a good sense about numbers, though, that her guess was so close. Unfortunately, she couldn't see it.

Teacher: Ellen, I am noticing that you are using those tally marks again. How's it working for you?

Ellen: Okay, except these numbers are too big for me. Sometimes I miscount them or erase them. Then I have to start all over again!

Here was my opening.

Teacher: Well, I think you know more about these numbers than you think. I'm going to take a chance with you right now and push you into something that I know you are ready for. You know that if I thought you needed to do it with tally marks, I would let you go on with it this way. But I'm not going to let you do that. I'm going to write an equation that I think you can use to solve this problem.

I wrote 4 × 10 = ___ on her paper. She immediately told me the answer was 40. I acknowledged that I thought she knew some things about multiples of 10 that could help her solve problems like this. I asked her if she could make use of that to solve the problem. She said she wasn't sure. I decided to help her reinterpret the problem.

Teacher: Well, suppose there were only 40 trucks. How many would each of the 4 friends get then?

Ellen: 10.

Teacher: Then, how many more would you have to work with if there were 52 trucks to begin with?

Ellen: Let's see. There would be 12 more, so it would be . . . 3 fours, so each child would get 3 more. That would be 10 and 3, or 13. (She wrote 13 in the blank to complete the equation.)

52 ÷ <u>13</u> = 4

Ellen had even surprised me with her ease in figuring this out so easily. I knew I had been leading her a lot, so after a short "debriefing" about the strategy I was suggesting, I left her to see if she could make use of it on her own.

The next day I checked back in with her.

Teacher: How are you doing with those problems, Ellen?

Ellen: They're really hard for me.

Teacher: Did that strategy work very well for you?

Ellen: Not really, I am pretty stuck.

Teacher: Let's take a look.

Sam was having a birthday party, and he was inviting 7 people to the party. He had 42 things for goody-bags. How many things will each child get?

42 ÷ ___ = 7

10 10 10 10 2

It was at this point that I realized what had happened. Her work on this problem showed her first attempt, but she could not go any further. I realized that using 10s would, of course, only work when the product was equal to or greater than the known factor × 10. Giving 10 each to 7 people would equal 70 things, way more than the 42 things that Sam had for the goody-bags. Because I had made the strategy about 10s, I had failed to help Ellen think more generally in terms of using the biggest chunk she could manage times the known factor and then see how the leftovers could be divided up by that factor.

I decided to move to the next problem.

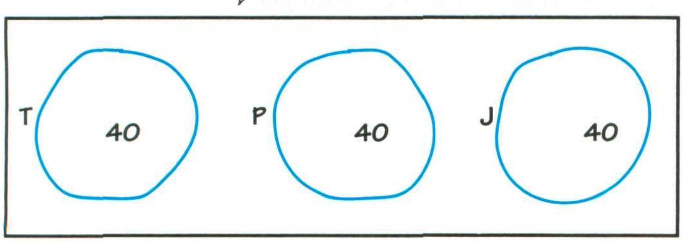
Tom, Paul, and Joe have to split a deck of 120 cards. How many cards do they each get? Equation: ____________

I asked her to write the equation, using the missing factor notation this time instead of the division sign. She wrote

$$\_\_\_\_ \times 3 = 120$$

Then I asked her to say the problem in her own words and to tell me what picture she had in her head for what was happening in the problem. Satisfied that she was ready to move to a solution, I asked her to make three circles to represent each person's share of the cards. She labeled each circle with the first initial of one of the people. I asked her to say the biggest number of cards she thought she could deal out to each. Her first guess was 40, and I asked her to write that number (not tally marks!) in each circle. When I asked her what she wanted to do next, she added 40 + 40 = 80, 80 + 20 = 100, 100 + 20 = 120; then she gasped, recognizing that she had figured it out! The figure below shows her final work.

Equation: $40 \times 3 = 120$

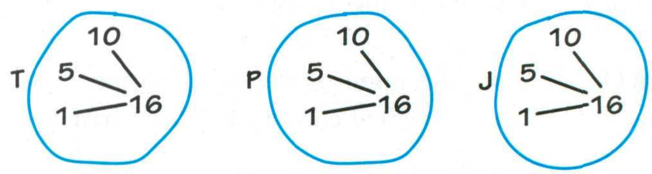


I decided to keep going. We proceeded similarly for the next problem.

Emma got 48 pieces of gum. Two friends came over. Emma wanted to share it with her friends. How many pieces would each person get?

I decided to comment only when she asked for it, when I was confused about something she did, or when I wanted to hear how she would explain the move she made. The only time I intervened was after she finished dealing out the first three 10s, and she thought she should try for another 10 for each friend. When she said she knew that would be too much, I asked if there was another chunk she would be able to use easily. From that point on, she worked independently toward the solution shown here:

Equation: $16 \times 3 = 48$

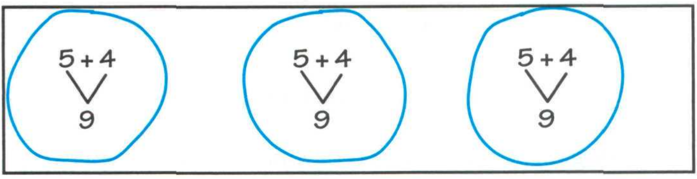


The next problem would be a real test of how confidently and competently Ellen would be able to make use of the strategy.

Sara is collecting cans and bottles for her community service project. She already has 27 cans and bottles, and she is putting them into 3 bags. How many will be in each bag?

Ellen started out by putting 12 cans or bottles in each bag, but after 2 bags, she stopped herself, and immediately went to putting just 5 into each bag. She did the problem completely on her own and felt successful continuing with the remaining problems.

Equation: $9 \times 3 = 27$



For Ellen, the nudging worked! It always feels like such a risk for me as the teacher to ascertain when there is an opening for me to intervene in this way. I felt that, for this student at this time, the nudge was necessary and well timed. I regret that my first attempt misled her, but I was grateful that she was open to returning to it again a few days later.

One thing I continue to learn is to trust the student. If I have misjudged what Ellen is ready for, she will not be able to make use of it, and ultimately that is what matters most. I have also learned how to read the student's reaction to my nudging. If I have moved Ellen too quickly from comfort to challenge, she may need more support working through ideas for herself at the place where she is comfortable before I can intervene with the next steps.

Ms. Sajak recognized that Ellen had strengths as a math learner, but she was anxious about math. She initially focused on observing Ellen's learning style and establishing a trusting relationship with her. She noted Ellen's progress as she learned to use multiples of 10 to solve addition and subtraction problems during the class work with the number line.

Later in the year, when Ms. Sajak noticed that Ellen was becoming frustrated as she attempted to use tally marks to solve multiplication and division problems, she decided to intervene and explicitly teach Ellen a more efficient strategy based on what Ellen understood about number relationships. This was a thoughtful decision on Ms. Sajak's part and was made only after determining that Ellen's understanding of equal groups and multiples would allow her to make sense of a new approach. Ms. Sajak modeled the strategy and worked with Ellen in her first attempts putting it into practice. Together they were able to work through the initial struggles of incorporating a new strategy.