Teacher Note

Why Study the U.S. Conventional Algorithms?

In *Investigations,* the algorithms traditionally taught in the United States are studied by students after they have developed their own fluent methods for solving problems with whole numbers in each operation. These include the algorithms for addition, multiplication, and subtraction which involve regrouping the numbers. Historically, these algorithms were developed for doing calculations by hand with a minimum of steps and compact notation. The power of these algorithms for quick calculation lies largely in the fact that they require the user to carry out a series of mostly single-digit calculations. They were designed so that the user could rely on a small set of known number combinations and the repetition of a small sequence of steps to solve any problem. These algorithms, as human inventions, are elegant and efficient.

However, in the elementary grades, when we want students to acquire solid understanding of the base-ten number system and the meaning of arithmetic operations, these algorithms tend to obscure both the place value of digits and fundamental properties of the operations. Research and practice in the field of mathematics education have shown that there are alternative algorithms and strategies that students develop, that help them maintain a focus on understanding place value and the operations and, at the same time, are easily generalized and efficient.

Although each student may primarily use one strategy for each operation, in *Investigations,* students are expected to study more than one algorithm or strategy for each operation. Students study a variety of approaches for the following three reasons:

- Different algorithms and strategies provide access to analysis of different mathematical relationships.
- Access to different algorithms and strategies leads to flexibility in solving problems. One method may be better suited to a particular problem.
- Students learn that algorithms are "made objects" that can be compared, analyzed, and critiqued according to a number of criteria.

As the NCTM's *Principles and Standards for School Mathematics* (2000) states:

> Many students enter Grade 3 with methods for adding and subtracting numbers. In Grades 3–5, they should extend these methods to adding and subtracting larger numbers, and learn to record their work systematically and clearly. Having access to more than one method for each operation allows students to choose an approach that best fits the numbers in a particular problem. (page 155)

In students' study of calculation methods for each operation, they first build strategies that they are comfortable with, that make sense to them, that they can use fluently, and that can gradually be applied to harder problems. At a later time they study some of the strategies they are less comfortable with in order to learn about the underlying mathematical relationships. This later period includes a study of conventional algorithms that are commonly used in the students' communities. This study of conventional algorithms has both a mathematical and a social purpose.

Students with good understanding of an operation—what it is used for, what its properties are, how to solve a problem that requires that operation efficiently, how it is related to other operations, and how the base-ten number system is used in that operation—can use a study of *any* algorithm that has been invented for that operation as an opportunity to delve further into the operation itself. Studying how and why an unfamiliar algorithm works is a challenge to think through what we know about an operation. It requires pulling apart an algorithm, bringing meaning to shortcut notations, and finding parts of the algorithm that are similar to parts of more familiar algorithms.

For example, one of the authors once noticed an older relative using the following algorithm to solve subtraction problems:

$$\begin{array}{r} 7\,{}^{1}5 \\ -\,{}^{4}39 \\ \hline 36 \end{array}$$

This woman had been educated in the United States in the early part of the twentieth century when this algorithm had been "standard" in some places in the country. By thinking through what this shortcut notation means, we can see that adding 10 to each number in the original problem, (70 + 5) − (30 + 9), gives us an equivalent problem, (70 + 15) − (40 + 9), that can be solved by subtracting each place. The underlying principle is that changing the two numbers in a subtraction problem by adding (or subtracting) the same amount results in a problem with the same answer as the original problem. Thinking through *why* an algorithm works brings us back to fundamental ideas about the operations, ideas on which these algorithms are based.

Another mathematical reason for studying these algorithms is that they have been used and found useful by many people. Too often in the past, these algorithms were taught and learned without meaning. And, too often, these algorithms were seen as the central teaching tool for learning about an operation: learning addition was defined as learning the steps of the "carrying" algorithm. However, whereas the "carrying" algorithm may have held an inappropriately central place in our teaching strategies at one time, it is a perfectly good algorithm that can be used by those who find it useful. Competent adults often use different algorithms for different contexts, use a mixture of algorithms, or use one algorithm or strategy to check another. For example, one of the authors has a particular algorithm for subtraction that she uses only in her checkbook (it is neither the standard borrowing algorithm nor any of those used in this curriculum)—it is one that she has shaped to fit her particular needs in that context. Therefore, a second reason for studying the carrying and borrowing algorithms is to provide students exposure to these algorithms and their underlying meaning. Those who find them sensible and useful may choose to adopt them for their own uses in life.

The third reason for studying conventional algorithms is that they are a part of the social knowledge in students' communities. Adults in students' lives may use these algorithms, and they need not be a mystery to students. Because a variety of algorithms have been taught in different countries and at different times in the U.S. (as, for example, the subtraction algorithm shown above), we recommend that you have students bring in algorithms used by adults in their families. You may find that there is more than one algorithm commonly used in the students' community for a particular operation.

The following are two primary goals for the study of numbers and operations in the elementary grades:

1. Understanding the meaning and properties of the operations

2. Attaining computational fluency with whole numbers

These goals underlie the choices we make in the study of algorithms and strategies. As stated in NCTM's *Principles and Standards:*

> Students should come to view algorithms as tools for solving problems rather than as the goal of mathematics study. As students develop computational algorithms, teachers should evaluate their work, help them recognize efficient algorithms, and provide sufficient and appropriate practice so that they become fluent and flexible in computing. (page 144)