Learning the Addition Combinations

To develop efficient computation strategies, students need to become fluent with the addition combinations from 1 + 1 to 10 + 10. Fluency means that the combinations are quickly accessible, either because they are immediately known or because the calculation is so effortless as to be automatic (in the way that some adults quickly derive one combination from another; for example, thinking 8 + 9 = 8 + 10 − 1). In *Investigations,* all students should be fluent with all of the addition combinations up to 10 + 10 by the end of Grade 2. However, some students may need to review and practice some of these combinations in Grade 3.

Why Do We Call Them *Combinations*?

The addition problems from 1 + 1 through 10 + 10 are traditionally referred to as "addition facts." The *Investigations* curriculum follows the National Council of Teachers of Mathematics (NCTM) convention of calling these expressions *combinations* rather than *facts* for two reasons. First, referring to *only* particular addition and multiplication combinations as *facts* seems to give them elevated status. This makes them seem more important than other critical parts of mathematics.

In addition, the word *fact* implies that something cannot be learned through reasoning. For example, it is a fact that the first president of the United States was George Washington, and it is a fact that Rosa Parks was born in Alabama in 1913. If these facts are important for us to know, we can remember them or use reference materials to look them up. However, the sum of 7 + 8 can be determined in many ways; it is logically connected to our system of numbers and operations. If we forget the sum, but understand what addition is and know some related combinations, we can find the sum through reasoning. For example:

If we know that 7 + 7 = 14, we can add 1 more to get 15.

If we know that 8 + 8 = 16, we can subtract 1 to get 15.

If we know that 7 + 3 = 10, we can then add the 5 that is left from the 8 to get 15. (7 + 8 = 7 + 3 + 5 = 15)

The term *facts* does convey a meaning that is generally understood by some students and family members, so you will need to decide whether to use the term *facts* along with *combinations* in certain settings in order to make your meaning clear. Further, it does not seem appropriate to refer to the counterparts for subtraction and division as "combinations," because subtraction and division do not involve the action of combining. Therefore, for convenience we refer to "subtraction facts" and "division facts."

Learning the Addition Combinations Fluently

The *Investigations* curriculum, like NCTM, recognizes the importance of students' learning the basic combinations fluently through reasoning about number relationships: "Fluency with whole-number computation depends, in large part, on fluency with basic number combinations—the single digit addition and multiplication pairs and their counterparts for subtraction and division. Fluency with basic number combinations develops from well-understood meanings for the four operations and from a focus on thinking strategies". . . . "[*Principles and Standards for School Mathematics,* pp. 152–153]"

In other words, students learn these combinations best by using strategies, not simply by rote memorization. Relying on memory alone is not sufficient. If you forget—as we all do at times—you are left with nothing. If, on the other hand, your learning is based on an understanding of numbers and their relationships, you have a way to rethink and restructure your knowledge when you do not remember something you thought you knew.

In Grade 2, students learned these combinations in groups (make-10 combinations; plus-1, -2, or -10 combinations; doubles and near-doubles), which helped them learn good strategies for solving them easily. Fluency develops through frequent and repeated use; therefore, as students worked on a particular category of combinations, they played games and engaged in activities that focused on those combinations. For example, students reviewed the combinations that make

10 by playing *Make 10* and *Tens Go Fish.* The Classroom Routine *Today's Number* provided another opportunity for practice.

Students in Grade 2 used Addition Cards to think about combinations they knew and to practice those they did not yet know. Over the year, students collected a set of Addition Cards for each category and sorted them into two envelopes: "Combinations I Know" and "Combinations I Am Still Working On." Students wrote clues on these cards to help them remember the combinations they found difficult.

In Grade 3, students again use Addition Cards (M24–M28) as they review the addition combinations. At the beginning of Investigation 2, they sort these cards as they did in Grade 2 and focus on the combinations they have not yet learned. As you observe your students and assess their knowledge of combinations later in Investigation 2, you will note that some may need more practice in one or more of these categories, particularly the final group of remaining combinations. Addition Combinations Practice (M29) contains blank addition cards for you or students to fill in, according to their individual needs.

Knowing the addition combinations should be judged not only by quick recall but also by fluency in use. Can students call on these combinations and use them easily as they solve other problems? Through repeated use and familiarity, students will come to know most of the addition combinations quickly. For the others, they will be able to use some quick and comfortable strategy based on reasoning about the numbers.

Categories of Addition Combinations

The categories of combinations are listed below. There are also notes about when most students learn these combinations. Note that some combinations fall into more than one category. For example, 1 + 9 and 9 + 1 is both a combination that makes 10 and a plus-1 combination.

Plus-1 and plus-2 combinations Many students leave Grade 1 fluent with the combinations that involve adding 1 or 2 to any single-digit number (8 + 1 and 7 + 2). As second graders come to understand that addition is commutative, they also become fluent with combinations in which the order of the numbers is reversed (1 + 8 and 2 + 7).

Make-10 combinations These two-addend combinations of 10 (e.g., 3 + 7, 4 + 6) were a benchmark for the end of Grade 1; students review them in Grade 2.

Doubles By the end of first grade, many students know their doubles combinations up to 5 + 5. In Grade 2, students work on these combinations up to 10 + 10. Students practice these combinations throughout Grade 2 and should gain fluency with them by the end of the year.

Near doubles (or doubles plus or minus 1) Students learn these combinations in Grade 2—those that are one more or one less than the doubles (e.g., 5 + 6, 7 + 8)—by relating them to the doubles.

Plus-10 combinations As students work on ideas about place value in Grade 2, they learn the plus-10 combinations—the sums of 10 and the numbers 1–10 (10 + 1, 10 + 2, 10 + 3, . . . 10 + 10).

Plus-9 combinations Students learn these combinations—the sums of 9 and the numbers 1–10 (9 + 1, 9 + 2, 9 + 3, . . . 9 + 10)—by relating them to the plus-10 combinations.

Remaining combinations Students who are fluent with doubles plus or minus 1 may be able to use the "clue" that several of the remaining combinations are doubles *plus or minus 2.* Students who are fluent with the make-10 combinations and with breaking numbers apart can solve most of these quickly (e.g., by breaking apart 7 + 5 into 7 + 3 + 2). Similarly, students can use their knowledge of make-10 combinations to solve "near-10" combinations (6 + 3, 7 + 4, 8 + 3).

Assessment: Addition Combinations

As you observe the students during the Addition Combinations assessment, you will find that a student is likely to fall into one of the three following groups:

Fluent Students who are fluent with their addition combinations to 10 + 10 can hear or read a problem, think for a moment, and then say the answer. Most students in Grade 3 should be in this category.

Nearly fluent Students in this category are fluent with many of these combinations but pause to figure out the answer to some ("8 + 5 is 8 . . . 9, 10, 11, 12, 13"). Note which combinations still cause trouble, and check that these match the cards in students' envelopes of "Combinations I'm Still Working On." Also, point them out to students: "You've come a long way with these combinations, but a few of them still seem to give you some trouble. How could we make it easier for you to remember that 8 + 5 and 5 + 8 equal 13?"

Continued practice with the activities described below will help these students become fluent. You can also use Addition Combinations Practice (M29) to assign students particular combinations to work on each week until they know them all.

Not yet fluent These students need to figure out many of these problems, using their fingers to count up or using cubes to model the problem. At Grade 3, there should be very few students in this category. They need more intensive practice with many or all of the activities described below.

Helping the "Nearly Fluent" Students

Students who are nearly fluent need more practice. They should use their Addition Cards to practice the combinations they are still working on. Work with them to write clues that they find helpful.

The categories listed below are likely to require further practice. Along with Addition Card practice, students will benefit from activities focused on these categories. Note that when students are relating one category of combinations to another, modeling the relationship between these combinations with cubes and on the number line is particularly important.

Near-doubles Arrange small-group work that helps students connect the near-doubles to the doubles combinations. Have them use cubes and the number line.

Plus-10 combinations Have students use concrete models to examine what happens when 10 is added to a number. Give students the opportunity to solve plus-10 problems by using the 100 chart, a number line, and cubes (with some in towers of 10).

Plus-9 combinations Arrange small-group work that helps students connect the plus-9 combinations to the plus-10 combinations. Have them use cubes and the number line.

Remaining combinations Arrange small-group work that helps students connect the remaining combinations to other categories (for example, relating the plus-8 combinations to the plus-9 and plus-10 combinations; the doubles plus or minus 2 combinations to the doubles; and combinations such as 3 + 6, 3 + 8, and 4 + 7 to the make-10 combinations). Have them use cubes and the number line.

Helping the "Not Yet Fluent" Students

Students who are not yet fluent may need practice with the above categories, as well as with some or all of the categories listed below. They can benefit from further experience with games and activities from the Grade 1 and 2 *Investigations* units. If you have access to these units, you will find the complete descriptions and recording sheets for each game or activity in the units named below. If you cannot get these, use the descriptions to design similar activities.

Many of these activities use the sets of Primary Number Cards, which, unlike the Digit Cards, include the number 10. Try to borrow or copy some sets of these cards, or consider using decks of regular playing cards with the jacks, queens, and kings removed.

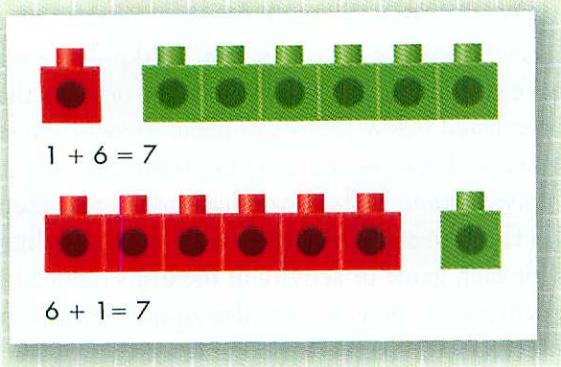
Plus-1 and plus-2 combinations The game *Plus 1 or 2 Bingo* (from Grade 2 *Counting, Coins, and Combinations*) provides practice with these combinations. Players have a 6×6 gameboard with the numbers 1–12 placed randomly in the squares. With the goal of covering an entire row, the student chooses a Number Card, adds 1 or 2 to that number, and covers that total. Including wild cards encourages students to work backward: "If I want to cover 8, what number should I make this wild card be?"

For many students, the most difficult combinations are those in which the first addend is 1 or 2 ($1 + 8$ or $2 + 7$). These students need time to make sense of why $8 + 1$ and $1 + 8$ have the same sum. You can work with a student or a small group to think about this idea. For example, pose two related story problems that use the same numbers. Have a collection of cubes or counters available for students to use to model what is happening from problem to problem.

I bought 1 red apple and 6 green apples at the store. How many apples did I buy?

The next day I went back and I bought 6 red apples and 1 green apple. How many apples did I buy that day?

Encourage students to explain their thinking and to use cubes to model their strategies.



"It's 6 and 1 and 1 and 6. It's still 7 altogether."

"It doesn't matter which order you count it in, it's still 7."

"You didn't add any more or take any away, so it's still 7."

For further practice, pose similar questions based on different contexts:

What if I were playing a game that involves rolling two number cubes? I rolled a 1 and a 5. How many altogether? What if I said I rolled a 5 and 1?

What about a game that uses the Digit Cards? I turned over two cards. I got an 8 and a 1. How many altogether? This time I got a 1 and an 8. Now how many?

The Combinations That Make 10

All of the following games and activities provide good practice with make-10 combinations.

Make 10 (from Grade 2, *Counting, Coins, and Combinations*) Cards are arranged in four rows of five, face up. Players take turns finding two cards that equal 10 when added together.

Tens Go Fish (from Grade 2, *Counting, Coins, and Combinations*) Each player is dealt five cards. Players take turns asking each other for cards that, when combined with a card in their hand, will total 10.

Today's Number: 10 (Classroom Routine in Grade 2) Students generate equivalent expressions for a particular number. Encourage them to use addition and only two addends.

How Many of Each? problems (from Grade 1, *How Many of Each, Solving Story Problems,* and *Number Games and Crayon Puzzles*) This recurring problem in Grade 1 is set up as follows: "You have 10 toys. Some are blocks, and some are marbles. How many of each could you have?" Give students these problems with different contexts (blocks and marbles, apples and oranges, peas and carrots) to provide additional practice.

Heads and Tails (from Grade 1, *How Many of Each?*) Students have a 2-column table labeled "Heads" and "Tails." They drop 10 pennies and record the number that land with heads and tails facing up. You can also use 2-sided chips or beans that have been painted two colors. Students can also play *On and Off,* recording the number that fall on and off a piece of paper.

How Many Am I Hiding? (from Grade 1, *How Many of Each, Solving Story Problems,* and *Number Games and Crayon Puzzles*) Make a tower of 10 cubes and show it to the student. Agree that there are 10 cubes. Then ask the student to close his or her eyes. Break off some cubes and hide them behind your back. Now show the student the cubes you did not hide. "How many am I hiding? How do you know?"

Counters in a Cup (from Grade 1, *Solving Story Problems* and *Number Games and Crayon Puzzles*) Show the student 10 counters (or chips, or pennies) and agree on the total. With the student covering his or her eyes, hide some of the items under a paper cup. "How many am I hiding? How do you know?"

Doubles

The following two activities are useful for practice with doubles:

Double It (from Grade 2, *How Many of Each?*) Students practice the doubles combinations by drawing a number card, doubling it, and recording the sum.

Doubles Arrays (from Grade 2, *How Many of Each?*) Students draw a Number Card (or Digit Card) and color in double that number of squares on grid paper. They record an equation that illustrates the double.