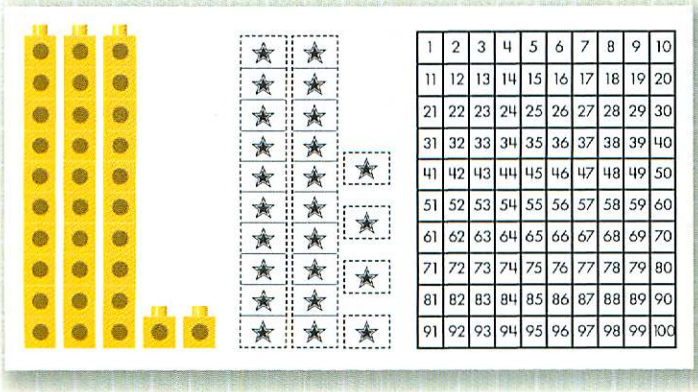
Mathematical Representations for Addition and Subtraction

As students work on addition and subtraction, it is important for them to develop visual images to help them make sense of problems, solve them, and represent the strategies they use. Students may develop their own images as they work on addition and subtraction. However, throughout the *Investigations* curriculum, they are also introduced to specific representations that they are encouraged to use as they solve problems.

In earlier grades, students used a variety of manipulatives and representations to make sense of numbers and operations. In this unit, use of the following math tools and representations is continued from earlier grades:

- interlocking cubes (stored in towers of 10 for ease of use and counting out how many are needed, and to reinforce the concept of 10 as the basis of our number system)
- 100 chart
- 200 chart (introduced in this unit)
- sets of coins and dollars
- class number line
- place value model: stickers as "singles" (units) and "strips" (tens)



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Two of these models are extended in this unit: the place value model and the number line.

The Place Value Model

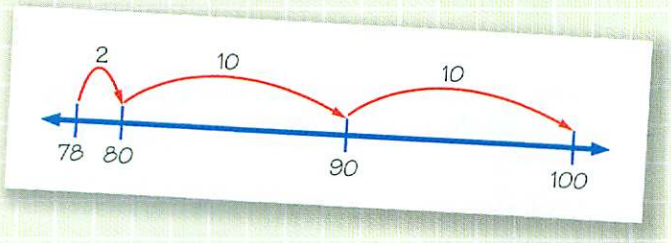
In Grade 3, the place value model with stickers is extended to include sheets of 100, which are shown as 10 × 10 arrays. This place value model highlights the structure of numbers in our base 10 number system. See *Implementing Investigations in Grade 3* for more about students' work with place value in this curriculum. The **Teacher Note:** Stickers: A Context for Place Value, page 145, explains how the sticker context supports this work. Students also use 100 grids (and sometimes strips of 10s and singles) as a representation for addition and subtraction in problems outside the sticker context.

As students solve addition and subtraction problems, they use this model to consider what happens when multiples of 10 are added to or subtracted from numbers. They also use this model to break numbers apart by place as they solve multidigit addition and subtraction problems. Students also use a 100 grid to visualize how a 2-digit number relates to 100, an important landmark in our number system. Later in Grade 3, they use this model to construct 1,000 and visualize how 2- and 3-digit numbers relate to 1,000.

The Number Line

In Grade 2, students used a class number line that was marked from 0 to 105. At Grade 3, a physical number line that extends from –20 to 120 in increments of 5 is displayed in the classroom. Students also learn how to represent their strategies on an unmarked number line, showing only those numbers that are relevant to the problem. As students add or subtract quantities in groups or chunks as well as by ones, they mark these moves with

curved lines and arrows or "jumps" on the number line. For example, for the problem 78 + ____ = 100, one student showed her work as follows:



Number lines help students better understand how a quantity increases or decreases, as some amount is added to or subtracted from it. When adding two numbers, students can visualize starting at the number that represents the first addend and adding the second addend by taking one or several jumps on the number line (see Figure A).

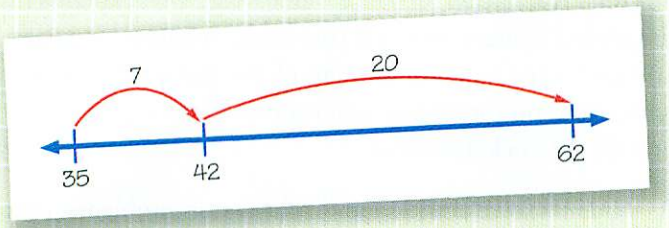


Figure A. *35 + 27 = 62*

They can visualize a subtraction problem either as the distance between the two numbers, or as a jump back on the number line. In the first case, the difference is represented by the jumps taken (see Figure B), so the answer is the distance between the two numbers on the number line. In the second case, the difference is represented by the number where the jumps end (see Figure C), and the jumps backward represent the amount being subtracted.

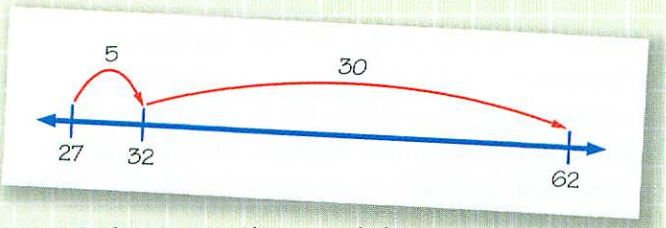


Figure B. *62 −27 = 35 (distance on the line)*

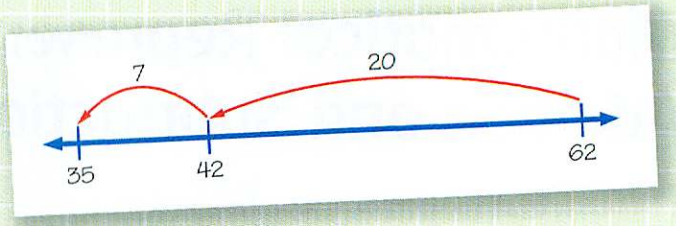


Figure C. *62 − 27 = 35 (where jumps end)*

Using a Variety of Models and Tools

Place value models and number lines help students to develop images about what is happening in problems, and to think about ways to solve them. They also highlight important aspects of our number system and number relationships. Therefore, using these representations helps students develop efficient problem-solving strategies based on their understanding of our number system.

In Grade 3, students expand their knowledge of math tools for whole-number computation. When asked to represent their thinking about an idea they are exploring, they can choose whichever math tool best captures how they are visualizing the number relationships and the action of the problem. Consequently, a variety of math tools and representations should be available at all times for students to use when they need to explain, verify, or further explore their thinking.

Students can be expected to use math tools both to solve problems and to represent their thinking. After students have become comfortable with a math tool or representation, they can begin to use it for mental reference. For example, when a student needs help solving a problem, you might say, "Imagine what this problem would look like on a number line—what do you see?" Over time, students internalize these tools and representations.