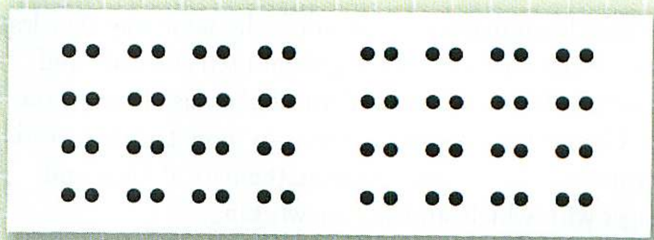
Finding Prime Factors

One way to categorize numbers is to examine their factors. From work in previous grades, students are familiar with *square numbers,* numbers that equal a whole number multiplied by itself. They are also familiar with *prime numbers,* numbers that have only two factors, 1 and the number itself. They know that some numbers, such as 36, have many factors, and others, such as 21, have few factors. They have had experience modeling factors and products by making rectangular arrays with tiles or on grid paper.

Most of students' past work has focused on finding pairs of factors for a given product. For example, ways to make 36 with two whole-number factors are: 1×36, 2×18, 3×12, 4×9, and 6×6. In this unit, students start with such familiar pairs of factors, and then build on these to find all the ways to multiply whole numbers to produce a certain product, using three, four, five, or more factors. For 36, the complete list is given below.

1×36	$2 \times 2 \times 9$	$2 \times 2 \times 3 \times 3$
2×18	$2 \times 3 \times 6$	
3×12	$3 \times 3 \times 4$	
4×9		
6×6		

When students work on *Quick Images* patterns, they find different multiplication combinations that represent the same array of figures, depending on how they picture the arrangements. For example, Problem 3 (shown in the next column) on *Student Activity Book* page 4, which shows 64 dots, can be described in a number of ways.



$(4 \times 2) \times 4 \times 2$
(4 groups of 2) × 4 rows × 2 sections

$8 \times 4 \times 2$
8 in each double column × 4 columns × 2 sections

16×4
16 in each row all the way across × 4 rows

8×8
8 in each double column × 8 double columns

Through visualizing representations such as this one and building on the pairs of factors they know, students generate as many multiplication combinations as they can (using whole numbers greater than 1) for a given product. See **Dialogue Box:** Multiplying with More Than Two Numbers, page 181.

Many fifth graders are fascinated with questions about factors. How many different ways can you find to multiply whole numbers to make 180? What is the longest combination you can find? Are there any other combinations that use the same number of factors? As students investigate these questions, they find that they can generate new combinations by "breaking up" ones they already know: if $2 \times 90 = 180$, then $2 \times (2 \times 45) = 180$. By working on such problems, students are calling on the associative property, a property they will examine formally in future years. It can be expressed in algebraic notation as $(a \times b) \times c = a \times (b \times c)$.

As they use one multiplication combination to generate others, students notice that at some point, they can no longer split up the numbers in their combinations into other factors. For example, while working with $2 \times 2 \times 3 \times 3 \times 5 = 180$, some students find that they cannot generate any multiplication combinations with more than five factors because 2, 3, and 5 are prime numbers. Prime factors cannot be split up into other factors (except by using 1). Some students also notice that, discounting different orders of the same numbers, there is only one "longest" multiplication combination for any number.

These students are beginning to notice an important property of whole numbers—that each whole number greater than 1 can be factored into a product of prime numbers in only one way. In other words, there is one and only one multiplication combination consisting of prime numbers for each whole number greater than 1. This principle, the Fundamental Theorem of Arithmetic, expresses the idea that every whole number can be uniquely named. The prime numbers in the combination for a given whole number are its prime factors, and the combination itself is called the *prime factorization* for that number. For example, the prime factorization of 30 is $2 \times 3 \times 5$, and the prime factors of 30 are 2, 3, and 5. The prime factorization of 90 is $2 \times 3 \times 3 \times 5$, and its prime factors are also 2, 3, and 5. (The prime factor 3 is used twice in the prime factorization of 90.)

In this unit, students are challenged to find the "longest way to multiply," using whole numbers. Fifth graders can find the longest combination for given products, but do not yet necessarily understand that there is one such unique combination for each whole number. It is not necessary to stress that some factors are prime numbers or to emphasize the term *prime factorization* for fifth-grade students.

Goals for the work on factors in this unit include the following:

1. Engagement and interest in noticing properties of number
2. Reasoning about number relationships (e.g., using a known multiplication combination, such as 3×60, to find equivalent combinations, such as $3 \times (20 \times 3)$)
3. Increasing flexibility in mental computation

This work also develops building blocks for future work with prime factors in computation with fractions and in algebra.