The Case of Kyle and Asiah: Multiplying by Multiples of 10

Sarah Peck is a math resource teacher. In this case, she works with a group of fourth graders who are stuck using inefficient multiplication strategies. Realizing that their lack of understanding about a major mathematical idea is holding them back, she provides a learning experience that enables them to work through the roadblock. Following the lesson, she notices a change in the students' mathematical understanding as well as their attitude about math.

I work with a small group of fourth graders once a week. Kyle has been very resistant to trying to understand the mathematics behind our work, and Asiah is constantly distracting others and is easily distracted herself. It is difficult to work with them in a group, even a small group, as they have been reluctant to participate or work to make sense of the mathematics we do. I am convinced that their reactions come from the gaps in their mathematical understanding and the feelings of inadequacy that result. Indeed, last week Asiah referred to herself as stupid when she had trouble with a problem.

Their class was working on double-digit by single-digit multiplication. Kyle and Asiah both showed an understanding of multiplication and were able to solve the problems, but they didn't have efficient strategies. On her homework, Asiah used repeated addition most of the time. Kyle broke the numbers into tens and ones but didn't multiply by *multiples* of 10. For example,

Asiah's Strategy		*Kyle's Strategy*	
$6 \times 32 = 192$		$6 \times 32 = 192$	
32	32	$6 \times 10 = 60$	
32	32	$6 \times 10 = 60$	
+32	+32	$6 \times 10 = 60$	
96	96	$6 \times 2 = 12$	
$90 + 90 = 180$		60	
$6 + 6 = +12$		60	180
	192	+60	+ 12
		180	192

Since I was unsure what they understood about multiplying by multiples of 10, I designed a lesson to try to find out.

I started by asking them to construct the 7s table on their papers. As they finished, I put a column of problems with 70 on their papers:

$7 \times 1 = 7$	$70 \times 1 =$
$7 \times 2 = 14$	$70 \times 2 =$
$7 \times 3 = 21$	$70 \times 3 =$
$7 \times 4 = 28$	$70 \times 4 =$
$7 \times 5 = 35$	$70 \times 5 =$
$7 \times 6 = 42$	$70 \times 6 =$
$7 \times 7 = 49$	$70 \times 7 =$
$7 \times 8 = 56$	$70 \times 8 =$
$7 \times 9 = 63$	$70 \times 9 =$
$7 \times 10 = 70$	$70 \times 10 =$

I asked them to solve as many of the 70s problems as they could in their heads, just writing down the answers. I told them they could use paper and pencil or a calculator when they got stuck. Calculators were also available for checking answers.

Teacher: How are the two columns different from each other?

Kyle: You're just doing 70 instead of 7, so you put a 0 on the answer.

Teacher: Why do you put on a 0?

Kyle: That's how it works!

Asiah: [nodding in agreement] You're not supposed to say you're adding a 0 because you're not really adding. So you just say you're putting on the 0.

Teacher: Well, it's certainly getting you the right answers! I wonder why it works. How does putting a 0 on the 7 change it?

Kyle: It makes it a 70.

Teacher: What's happened to the 7 part?

Kyle: It's now 7 tens.

Teacher: Exactly! It went from being 7 ones to being 7 tens. We've actually multiplied the 7 by 10 when we do that.

Kyle: Is that what happens when you put on the 0? Are you always multiplying by 10?

We tried various numbers on the calculator to see if it worked with large numbers too, and it did. This was new information.

We looked back at the 70s problems to see what else we could discover.

Asiah: 70 × 6 is just like 7 × 6 but with a 0.

Teacher: Let's start with 7 × 6 = 42. When I change it to 70 × 6, what have I done to the 7?

Kyle: You times the 7 by 10.

Teacher: So what will I have to do to the answer [42]?

Kyle: [after a long pause] Times the 42 by 10?

Asiah: Yes, it's going to be 420.

Teacher: Whatever I do to change the problem, I also have to do to change the answer to keep them equal. [I make a mental note that this is another idea we need to explore more.]

Next, I give them four challenge problems to work on. Calculators are available for checking but not for solving. The four challenge problems are as follows:

3 × 70 = ____
30 × 7 = ____
70 × 60 = ____
28 × 7 = ____

They solved them independently and then shared their ideas. For a change, they were really listening to each other. They both solved the first two using the basic factor pair (3 × 7) and putting a 0 on the end. There was some confusion about the third problem (70 × 60), so we broke it into steps:

Teacher: What fact will you start with?

Asiah: 7 × 6 = 42

I wrote it on the board.

Teacher: How does the 7 change?

Asiah: You multiply it by 10 to get to 70.

I changed the 7 to 70 on the board.

Teacher: How will that change your answer?

Kyle: It's 420.

I changed the 42 to 420 on the board.

Teacher: How does the 6 change?

Kyle: You times it by 10.

I changed the 6 to 60 on the board.

Teacher: How will that change your answer?

Kyle: [his face lights up] Oh, I get it. It will be 4,200. That's easy!

Asiah: I love this because we can do hard problems.

Teacher: Sometimes if you break the problem apart and think about it in steps, it does make the problem easy to solve. How can you break the last problem [28 × 7] apart to make it easy to solve?

Asiah isn't sure, but Kyle breaks it into tens and ones as he did on his homework. This time, however, he doesn't break 20 into 2 tens.

Kyle: You just break the 28 into 20 and *8 and times* both by 7! That's really easy . . . except I have to figure out 8 × 7. Oh wait, I already did. It's right here. [He points to the 7s table he constructed at the beginning of the lesson.]

We tried a couple more two-digit by one-digit problems. They were excited by how easy they were to solve.

Teacher: So if you had your homework problems to do now, would you do them differently?

Kyle: You bet!

I gave him the six homework problems to do over again and he whipped through them, using his new idea about multiplying by multiples of 10. Asiah used a different strategy for the first problem, 8×22. She solved 4×22 and doubled it. This was the same strategy she used on her homework, and it is a good one for this problem. However, I asked her if she could also do it another way, and she broke the 22 apart into 20 and 2 and solved it correctly that way.

When they finished redoing their homework, they asked me to give them more hard problems. When our time was up, they begged to stay. When I said that wasn't possible, they asked if they could come during recess. Who are these new math lovers? Certainly not the Kyle and Asiah I've had before. They did come during recess and continued working on one-digit by two-, three-, even four-digit problems. They felt empowered and smart and couldn't wait to show their work to their classroom teacher and their parents.

As we wrapped up at the end of recess, we talked briefly about what made these problems easy.

Kyle: You just have to know how to multiply by 10.

Teacher: Well yes, but you also need the factor pairs we have been learning, like 7×6.

Asiah: But you don't have to learn everything. Like you don't have to *learn* 37×6. You just break the 37 into 30 and 7.

We made a major breakthrough today, not only in the students' understanding of the math, but also in their attitude about math. Asiah and Kyle are beginning to see that math can make sense, or rather that *they* can make sense of math. They *want* to practice, to solidify what they are beginning to understand about multiplying by multiples of 10. They are focused, listening to each other, and taking risks. Wow! What a difference!

What caused this change? I have to remind myself that although it is December, I have only met with them ten times. We are still getting comfortable with each other. This was only the second time we worked on multidigit multiplication. I was able to identify a big mathematical idea holding them back. They understood multiplying by 10 at some level, but they didn't know how to use that information in their solving strategies. They also didn't really understand what putting a 0 on the end meant or how they could expand the idea to multiplying by multiples of 10. As they looked across their threshold of understanding, they saw something powerful that was within their grasp. They wanted to own it.

I have no illusions that they've mastered this idea, but they are certainly in a different place than they were last week. I'm sure they will move in and out of using multiples of 10 in their strategies as they come to own the idea. How will they approach tonight's homework?

Realizing that two of her students were stuck using inefficient multiplication strategies, Ms. Peck created a learning experience that gave the students a chance to experience the power of multiplying by tens. Now able to see past the notion of "you just add a zero on the end," the students approached multiplication with both excitement and a newfound confidence.

Questions for Discussion

1. **What did Kyle and Asiah each understand about multiplication at the beginning of this case? What did they not understand?**
2. **How did learning about multiplying multiples of 10 help each of them move toward more powerful strategies for solving multiplication problems?**
3. **What commonalities do you see between Ms. Peck's work with Kyle and Asiah and Ms. Lloyd's work with Maura in the previous case?**
4. **Think about one or two of your students who are struggling and also lack confidence. How can you build their confidence while offering them experiences to improve their understanding?**