# Teacher Note

## Representations and Contexts for Mathematical Work

Mathematics involves describing and analyzing all kinds of mathematical relationships. Throughout the *Investigations* curriculum, students use representations and contexts to help them visualize these mathematical relationships. Thinking with representations and contexts allows students to express and further develop their ideas and enables students to engage with each other's ideas. Whether solving a multiplication problem, finding the area of a rectangle, describing the relationship between two variables, or ordering fractions, students use representations and contexts to investigate and explain.

The *Investigations* curriculum introduces a limited number of carefully chosen representations and contexts because they provide representations of mathematical relationships that students can use to solve problems and/or to show their ideas and solutions to others. Students may first use representations or contexts concretely, drawing or modeling with materials. Later, they incorporate these representations and contexts into mental models that they can call on to visualize the structure of problems and their solutions. Students develop the habit of making drawings, building models, and using representations to think with and to explain their thinking to others. They develop a repertoire of representations that they know well and can apply when faced with unfamiliar problem situations.

Good contexts and representations have the following characteristics:

- They are useful for a whole class of problems (e.g., addition problems).
- They can be extended to accommodate more complex problems and/or students' expanding repertoire of numbers.
- They do not overwhelm or interfere with the focus on mathematical content.
- Their structure embodies important characteristics of the mathematical relationships.

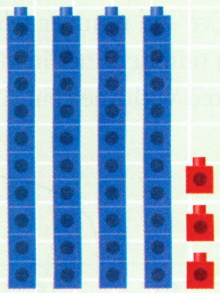
This Teacher Note provides some examples of how models, materials, and contexts are used by students across the grades.

### Representations

Basic representations in the *Investigations* curriculum include connecting cubes, the 100 chart (and its variants, the 300, 1,000, and 10,000 charts), number lines, arrays, and sets of two-dimensional (2-D) and three-dimensional (3-D) shapes. Each representation provides access to certain characteristics, actions, and properties of numbers and operations or of geometric properties and relationships. Here are two examples.

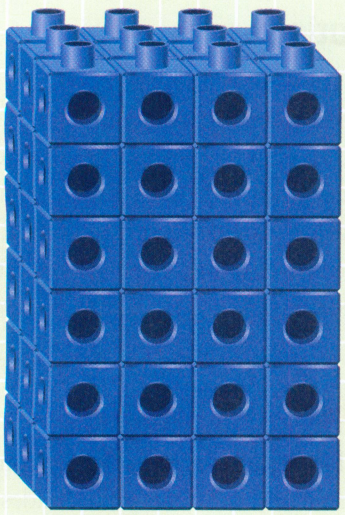
#### Connecting Cubes

Connecting cubes are a basic material for counting and for modeling addition and subtraction in Grades K–2. The cubes are a discrete model of whole numbers and provide a uniform counting material for representing ones. Because they connect, they can be organized into sticks of ten cubes so that students can use them to represent tens and ones.



The individual cubes are visible in the connected stick of ten, so students can visualize how this stick represents the equivalence of 1 ten and 10 ones and then how 10 ten-sticks is equivalent to 1 hundred and 100 ones. Connecting cubes are a flexible material. They are well suited for modeling the basic actions of joining and separating. They can also be used

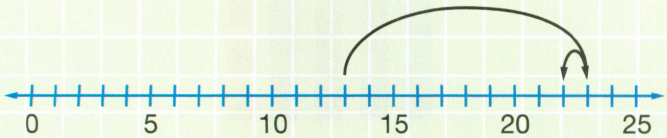
to construct rectangular arrays for studying multiplication and area. Students also use the cubes to construct rectangular prisms and to analyze and visualize how the volume of the shape consists of a certain number of layers, each of which has the same dimensions.



*Each layer is 3 × 4. There are six layers.*

## The Number Line

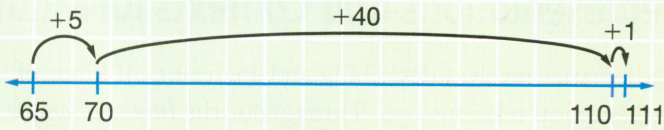
The number line is another key representation of numbers. This continuous representation offers students another view of the number sequence and number relationships. Students' beginning work with number lines involves number lines that are already marked with the counting numbers.




13 + 9

*I jumped up 10 to 23, then back 1.*

Later, students choose the part of the number line they need and which points on it should be marked as they use it to solve problems.



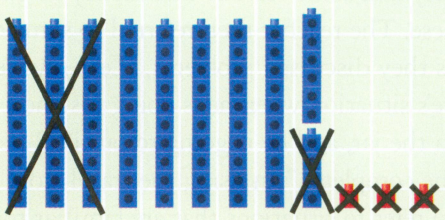

*65 + 46 = 111*

The number line provides access to the idea that numbers are infinite. At first, students come to this idea in relation to the counting sequence of whole numbers. Later, as they encounter negative numbers, they consider how the number line extends in both directions, that both positive and negative numbers "go on forever." In their study of rational numbers, they use the number line to model fractions and decimal fractions and consider how the segments of the number line between two successive whole numbers can be divided into smaller and smaller pieces. In later years, they will come to understand that there are an infinite number of numbers between any two successive integers.

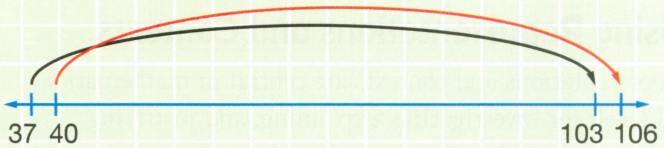
For students to use a representation well, they need enough experience with it so that they understand its basic characteristics and can then use it themselves to model and solve problems. For example, using an unmarked number line flexibly requires that students have enough prior experience using the marked number line to count and add.

## Using Different Representations

Different representations offer different models of the mathematics and access to different mathematical ideas. For example, both place value models and number lines are useful in students' study of subtraction, but they each allow students to see different aspects of subtraction. A student solving the problem 103 − 37 might think about subtracting 37 in parts by visualizing a place value model of the numbers, subtracting 3 tens and then 7 ones (which, for ease of subtraction from 103, the student might split into 3 + 4).



Another student might think about creating an easier, equivalent problem: 103 − 37 = 106 − 40. This student might visualize "sliding" the interval from 37 to 103 along a number line to determine how to change the numbers, while preserving the difference between them.




*103 − 37 = 106 − 40 = 66*

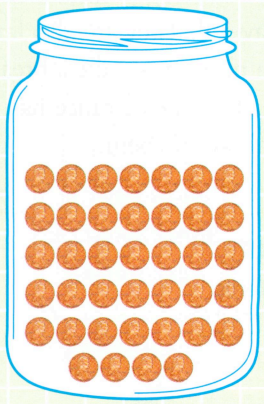
More details about these and other representations are provided throughout the curriculum units.

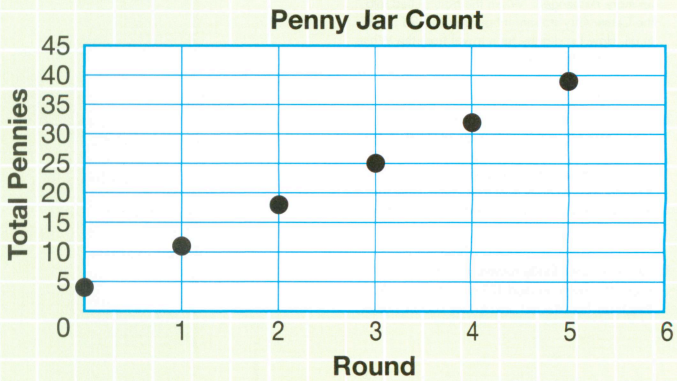
## Contexts

Contexts and stories are also used to represent mathematical relationships. A good context can be created from familiar events or fantasy. Contexts that students can imagine and visualize give them access to ways of thinking about the mathematical ideas and relationships they are studying. For a context to be useful, it must be connected enough to students' experience that students can imagine and represent the actions and relationships. At the same time, the details of the context need not be elaborate, so that the nonmathematical aspects of the context stay in the background. Here are two examples.

### The Penny Jar

One of the contexts in the patterns and functions units in Grades 1 and 4 is the Penny Jar. The Penny Jar contains some number of pennies (the starting amount) and then has a certain number of pennies added to it each day or with each round (the constant rate of change). This is one of the contexts used to engage students in exploring a function—the relationship of the number of days to the total number of pennies—that involves a constant rate of change. Students' knowledge of similar real-world contexts engages students quickly in the mathematics and helps them visualize the mathematical relationships, but it is not so elaborate that it obscures or distracts from the mathematics.



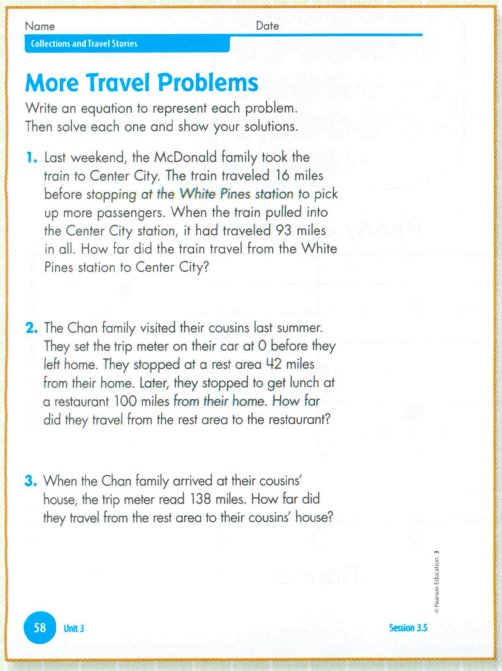



| Number of Rounds | Total Number of Pennies |
|---|---|
| Start | 4 |
| 1 | 11 |
| 2 | 18 |
| 3 | 25 |
| 4 | 32 |
| 5 | 39 |

Once students are familiar with the Penny Jar context, they can represent it in multiple ways, using pictures, tables, and graphs, to describe and analyze the relationship between the two variables.

### Travel Stories

In Grade 3, travel stories are used as a context for subtraction. Students are familiar with taking trips by car or bus or have encountered such trips in stories or movies. They know abouta trip having a starting point, an ending point, and a certain distance traveled. They are also familiar with stopping along the way for a meal or to take a break and with discussing how much of the distance has been covered and how much is still ahead of them.



Name Date

Collections and Travel Stories

**More Travel Problems**

Write an equation to represent each problem. Then solve each one and show your solutions.

1. Last weekend, the McDonald family took the train to Center City. The train traveled 16 miles before stopping at the White Pines station to pick up more passengers. When the train pulled into the Center City station, it had traveled 93 miles in all. How far did the train travel from the White Pines station to Center City?

2. The Chan family visited their cousins last summer. They set the trip meter on their car at 0 before they left home. They stopped at a rest area 42 miles from their home. Later, they stopped to get lunch at a restaurant 100 miles from their home. How far did they travel from the rest area to the restaurant?

3. When the Chan family arrived at their cousins' house, the trip meter read 138 miles. How far did they travel from the rest area to their cousins' house?





▲ Grade 3 Unit 3 *Student Activity Book*, page 58

### Helping Students Connect to Contexts

Teachers often personalize these contexts for students to help them visualize and use it. For example, when using the Penny Jar context, one second-grade teacher had a brief discussion about places they or someone else they know used to hold money and some reasons that money might get added to any of these. The teacher then referred to some of these situations as they discussed problems, "Let's say we're talking about Andre's situation when he is doing his chores. He has 3 pennies in the jar, and he is going to put in 2 pennies for each chore he completes." In using the travel story context, teachers also refer to situations that are familiar to students: "So let's say Janelle and her family are setting off to visit her grandma, like they did last summer, and the whole trip is 274 miles. …"

More details about these and other contexts are provided throughout the curriculum units.

## Using Representations and Contexts

Representations and contexts are central in mathematics at all levels for investigating, explaining, and justifying mathematical ideas. Students should move toward developing mental models of mathematical relationships that they call on routinely and will often use pictures, diagrams, and objects when they encounter new kinds of problems.

Students should use representations and contexts judiciously and with purpose. A first grader who is solving word problems that involve addition and subtraction might model every problem with cubes. Another student in the same class might model one or two problems; then, having visually confirmed the action of the operation, the student might solve the rest by imagining one quantity and counting on. A third student—or the same student later in the year—might reason about the numbers without using an image or model. In class discussions, both the teacher and students use representations to clarify and investigate mathematical ideas and to help all students focus on what is being discussed.

As a teacher, one of your roles is to support students in using representations and contexts and to help them develop mental images that they can call on. On the one hand, students need not show a picture for a problem when they have developed more efficient numerical tools and methods. For example, when one fourth grader was asked to solve a multiplication problem in two ways, he solved the problem by breaking it

up efficiently, using the distributive property, and then showed a solution using groups of tally marks. His teacher let him know that using tally marks was not what she was looking for from him and reminded him of the work the class had been doing on changing one of the numbers in the problem and then adjusting the product.

On the other hand, students should understand that the use of representations and models is not a "crutch" in mathematics but are a powerful set of tools for investigating problem situations. In the classroom, encourage representation as a central part of mathematics activity. Make a habit of asking questions such as these:

- Is there a way you can show us your thinking using the number line or the 100 chart?
- Can you explain how your strategy makes sense using the travel context we have been using for some of the problems?
- You used a number line and Chris used a place-value sketch showing tens and ones. What is similar or different about these two approaches? Where can you see the four tens in Chris's place value sketch on Luc's number line solution?
- Karen, you are thinking of the multiplication problem as representing 47 classrooms with 23 students in each class. How did this context help you keep track of the parts of the problem?
- Can you show us with a picture or on the Geoboard what you mean when you say "a triangle is half of a rectangle"?
- What if you needed to explain or prove what you are saying to someone who came to visit our classroom? Is there a way you can show me why what you are saying is true with a picture or diagram?

When students are accustomed to incorporating representations in their daily mathematics work and considering what representations can be helpful for explaining mathematical ideas, they can also create their own images appropriate to a particular problem situation. Help students make these images simple enough so that they serve the mathematics rather than obscure it. The use of representations in class discussions helps illuminate students' ideas for each other and, by putting out an image that is available to all students, clarifies what mathematical relationships are being considered and invites more students into the conversation.

For further examples of students' use of representations, see the classroom stories in the section "Language and Representation" in Part 7, Working with the Range of Learners: Classroom Cases.