Teacher Note

Multiplying by Multiples of 10

One of the traditional mathematical "tricks" that help with hard multiplication was this "rule": When you multiply by 10, just add a 0; when you multiply by 100, add two 0s; and so on. Many students notice that when they multiply a multiple of 10 such as 6×40, they can "ignore" the 0, multiply the other digits (in this case, $6 \times 4 = 24$), and then "add a 0" to get an answer to the original problem. In this unit, students consider what "adding a 0" means and why it works. Although this pattern is a very useful one, it is critical that students grasp the underlying mathematical relationships in order to develop a solid understanding of multiplying by multiples of 10.

Our number system is based on powers of 10: ones, tens, hundreds, thousands, and so forth. The value of a digit in each place in a number in this system is 10 times greater than the value of the same digit to its right.

$8 \times 1 = 8$ ones $= 8$

$8 \times 10 = 8$ tens $= 80$

$8 \times 100 = 8$ hundreds $= 800$

$8 \times 1{,}000 = 8$ thousands $= 8{,}000$

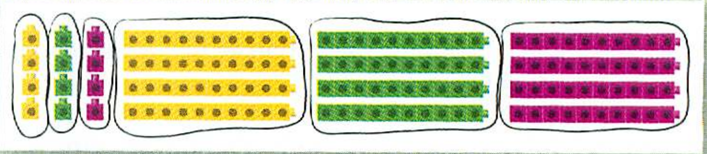
As the numbers in the last column increase, students see the addition of a 0 each time. In fact, what is happening is multiplication, not addition: Each number is 10 times greater than the previous number.

This progression becomes more difficult to see when multiplying multiples of 10, for example, 3×40. One way for you to think about how the product of 3×4 is related to the product of 3×40 is to look at the following series of equations:

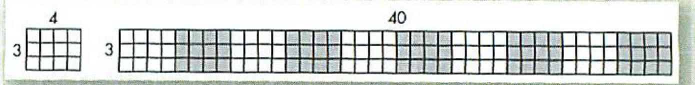
$$3 \times 4 = 12$$
$$(3 \times 4) \times 10 = 12 \times 10$$
$$(3 \times 4) \times 10 = 120$$
$$3 \times (4 \times 10) = 120$$
$$3 \times 40 = 120$$

This series demonstrates, through use of the associative property, that multiplying (3×4) by 10 is equivalent to multiplying 3×40. Therefore, the product of 3×40 is 10 times the product of 3×4.

Students are not likely to understand the relationship between 3×4 and 3×40 in this way, although some of them may be able to articulate that 3×40 is 10 times 3×4 because 40 is 10 times 4. For students, using arrays and other representations to show this relationship provides the best opportunity for them to see how multiplying by 10 and by multiples of 10 works. Consider the following two representations:



In this first representation, there are 12 cubes arranged in 3 groups of 4 cubes each: $3 \times 4 = 12$. Next to it are 3 groups with 4 towers of cubes in each group. There are 10 cubes in each tower. So there are 3×4 or 12 towers of cubes. Because each tower has 10 cubes, the total number of cubes is 12×10, or 120. In the first picture, there are 12 ones, or 12; in the second, there are 12 tens, or 120.



The second representation uses arrays to represent this relationship. The 3×4 array shows 3 rows of 4 squares. Ten of these smaller arrays are put together to create the 3×40 array. Now there are 3 rows of (4×10) squares. The 3×40 array has an area 10 times as large as the 3×4 array.

By visualizing and discussing representations such as these, students can develop a sound basis for understanding the "add a 0" rule.