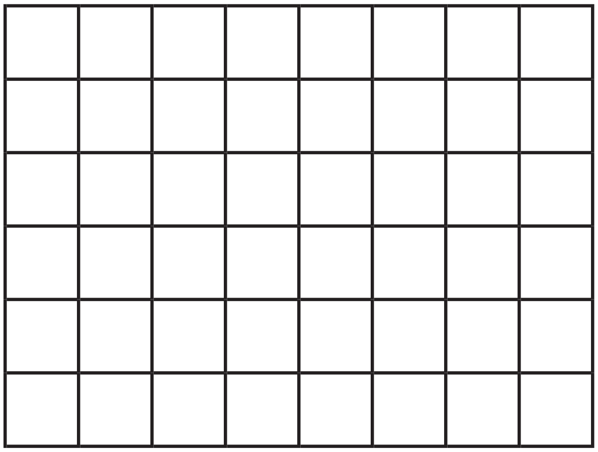
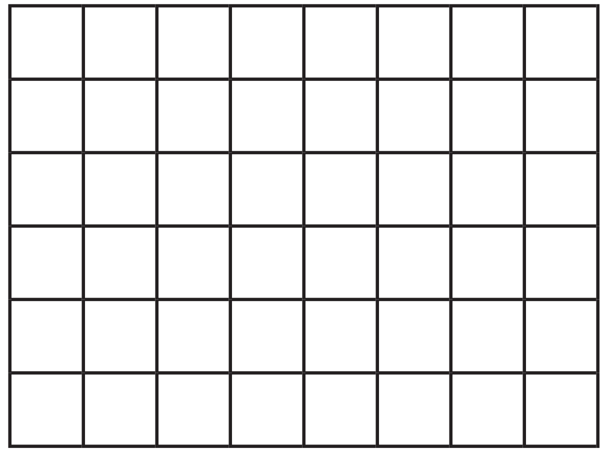
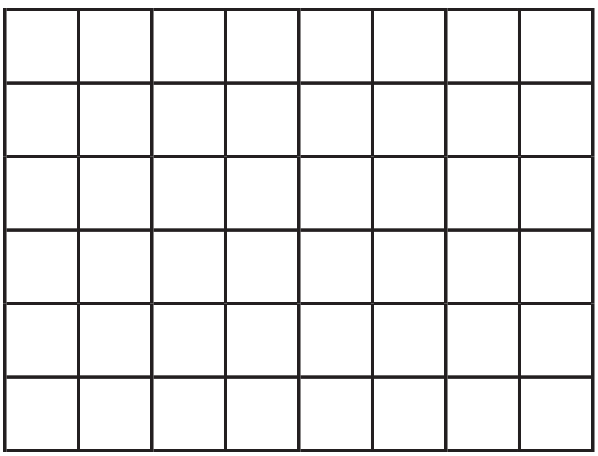
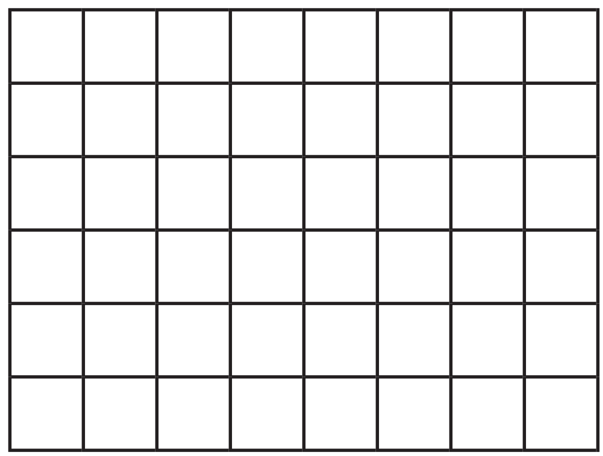
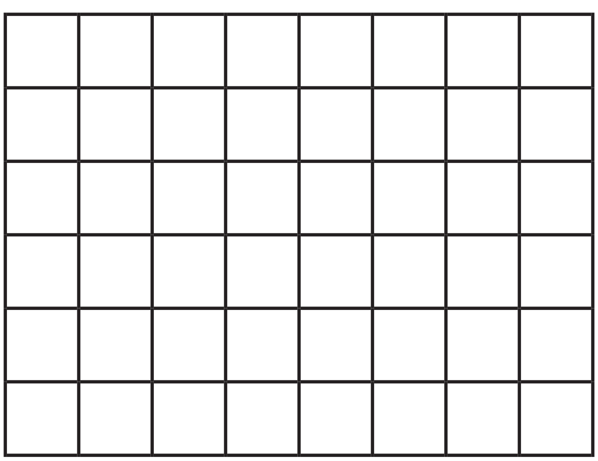
A 6 x 8 Array

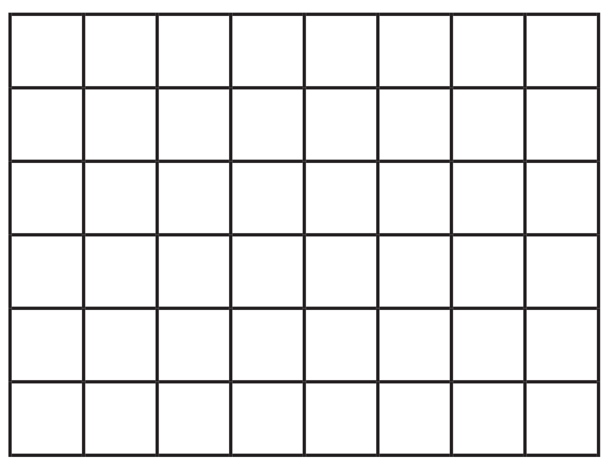












Video Notepad 1

Ryshawn, 29 x 4	Nicholas, 29 x 4

Jemea, 29 x 12	Jen, 89 x 12

summary of tape 2

Teaching and Learning About Multiplication and Division

After a brief introduction, this narrated videotape is presented in five segments: "Thinking About Equal Groups," "Learning About Factors," "Understanding How Multiplication Works," "Understanding How Division Works," and "Conclusion." A summary is provided only for the student work to be discussed during Session 3.

Understanding How Multiplication Works

This segment of the tape shows two third graders explaining how they would find "how many legs are on 29 elephants," followed by three fifth graders solving two-digit multiplication problems.

RYSHAWN 29×4

$25 + 25 + 25 + 25 = 100$

$4 + 4 + 4 + 4 = 16$

$29 \times 4 = 116$

NICHOLAS 29×4

$20 \times 4 = 80$

$9 \times 4 = 36$

$80 + 36 = 116$

JEMEA 29×12

$30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 = 360$

$360 - 12 = 348$

JEN 89×12

$89 \times 10 = 890$

$890 + 100 + 100 = 1090$

$1090 - 10 - 10 = 1070$

$1070 - 1 - 1 = 1068$

THOMAS 36×17

I added 4 to 36 to make it 40, and I added 3 to 17 to get to 20.

$$\begin{array}{r} 40 \\ \underline{\times 20} \end{array}$$

So I timesed 40 times 20 to get up to 80 [*he likely means 800*].
I knew it wasn't the answer, so I minused 4 because I added 4 to get up to 40, and that brought me to 796.
And I minused 3 'cause I had to add 3 to get to 20, and I got the answer 793.

Video Notepad 2

Thomas, 36 x 17

Teacher Note

Multiplication Strategies

On the basis of the first step students take, their strategies for multiplication fall into the following 3 basic categories:

1. Breaking the numbers apart by addition
2. Changing one number to make an easier problem
3. Creating an equivalent problem

In the first strategy, students break the given numbers apart and create a series of partial products. In the second and third strategies, students change the problem in some way to create a problem that is easier to solve. Students often use a combination of these approaches when solving a single problem and may see their own variations or combinations as different strategies.

In order to use the strategies described in this **Teacher Note,** students need to understand the meaning of multiplication and have a good mental model of what is happening in the problem. They need to look at the problem as a whole, think about the relationships of the numbers in the problem, and choose an approach that they can carry out easily and accurately.

Using the distributive property is essential in solving multiplication problems. By using this property, students can break the numbers in any multiplication problem into parts and then multiply each part of one number by each part of the other number(s). For example, if the problem is 48 × 42, they might think of 48 as 40 + 8 and/or think of 42 as 40 + 2. This breaking apart is usually done by place but not always. For example, for the problem 27 × 18, the student might think of 27 as 25 + 2.

It is not necessary for students to use the term *distributive property* or the notation (40 + 8) × (40 + 2). What is important is that they realize that the numbers can be broken apart by addition, that they know how to keep track of multiplying the parts, and that they add all the partial products to find the final product.

At the end of Grade 4, students should be able to use one or more of these strategies and record their solutions clearly. Although they may use one strategy most of the time, they should be able to consider solving a multiplication problem in more than one way. Students in Grade 4 spend a great deal of time studying strategies that involve breaking numbers apart by addition and changing one number to create a problem that is easier to solve. They will encounter the third strategy, creating an equivalent problem, in Grade 5.

Here are examples of the 3 strategies:

1. Breaking the numbers apart by addition

Many students choose to break the numbers apart by place and find all the partial products. Here are 2 ways a student might record this approach.

48 × 42		48	
40 × 40 =	1,600	× 42	
		1,600	40 × 40
40 × 2 =	80	320	40 × 8
8 × 40 =	320	80	2 × 40
8 × 2 =	16	16	2 × 8
	2,016	2,016	

Note that because of the commutative and associative properties of multiplication and addition, numbers can be multiplied or added in any order.

For a simpler problem, students might break up only one number. For example, to solve 22 × 13, a student might break up only the 13, thinking of the problem as (22 × 10) + (22 × 3), because both of these partial products are solved easily.

from Grade 4, Unit 8: How Many Packages? How Many Groups?

2. Changing 1 number to make an easier problem

In this sample solution, a student changes 48 × 42 to 50 × 42, solves 50 × 42, and then compensates for the initial change.

$$48 \times 42$$

$$50 \times 42 = 2{,}100$$

$$2{,}100 - 84 = 2{,}016$$

Changing 48 × 42 to 50 × 42 results in a problem that is easier to solve. The new numbers can be either broken apart by addition, yielding (50 × 40) + (50 × 2), or thought of as $\frac{1}{2}$ of 100 × 42. Then the student has to decide how to adjust the answer to 50 × 42. Because 50 × 42 is 2 more groups of 42 than 48 × 42, subtracting 2 groups of 42, or 84, gives the final answer.

It should be noted that, although this strategy will always work, it does not always make the problem easier to solve. It is possible to solve a multiplication problem by changing *both* numbers to make an easier problem (e.g., changing 48 × 42 to 50 × 40), but it is difficult to figure out how to adjust that answer in order to solve the original problem.

3. Creating an equivalent problem

Here are 2 ways students might create an equivalent problem.

$$48 \times 42 = 96 \times 21$$

$$48 \times 42 = 16 \times 126$$

Students often call this strategy "doubling and halving" because that is how they most often create equivalent problems in multiplication. In these examples, however, you can see that the first student "doubled and halved," and the second student actually "tripled and took one third." Using the same strategy, the students also could have changed the problem to 24 × 84 or 144 × 14.

As with changing one number to create an easier problem, it makes sense to use this strategy only if it does indeed result in a problem that is easier to solve. Sometimes a series of steps of doubling and halving can result in a much easier problem, such as 48 × 42 = 24 × 84 = 12 × 168.

When a student's first step is to create an equivalent problem, the next steps often include a combination of the other strategies. After a student has changed the problem to 96 × 21, it can be solved by breaking the numbers apart: 96 × 21 = (96 × 20) + (96 × 1).

One way that you might want to think about how this strategy works is that one of the numbers is first broken apart by multiplication and then the associative property is applied. Two examples are shown below.

$$48 \times 42 = 48 \times (2 \times 21) = (48 \times 2) \times 21 = 96 \times 21$$

$$48 \times 42 = (16 \times 3) \times 42 = 16 \times (3 \times 42) = 16 \times 126$$

Students study creating equivalent problems in multiplication and division in more depth in Grade 5.

from Grade 4, Unit 8: How Many Packages? How Many Groups?