Two Kinds of Division: Sharing and Grouping

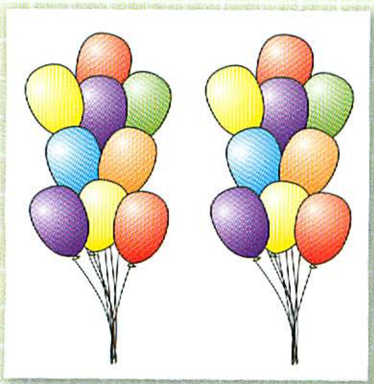
In this unit, students encounter two kinds of division situations. Consider these two problems:

I have 18 balloons for my party. After the party, I'm going to divide them evenly between my sister and me. How many balloons will each of us get?

I have 18 balloons for my party. I'm going to tie them together in bunches of two to give to my friends. How many bunches can I make?

Each problem is a division situation—a quantity is broken up into equal groups. The problem and the solution for each situation can be written in standard notation as $18 \div 2 = 9$. Yet these two situations are actually quite different. In the first situation, the *number of groups* of balloons (2) is given. The question is "How many balloons will be in each group?" In the second situation, the number of balloons *in each group* (2) is given, and the question is "How many groups will there be?" Each problem involves equal groups of balloons, but the results of the actions look different.

I have 18 balloons and two people. How many balloons does each person get?



I have 18 balloons to put into bunches of two. How many bunches will there be?

The solution to each problem is 9, but in the first problem 9 is the number of balloons per person (the number in each group). In the second problem, 9 is the number of bunches of balloons (the number of groups).

The first situation is probably the one with which your students are most comfortable because it can be solved by "dealing out;" that is, the action to solve the problem might be: one for you, one for me, one for you, one for me until all the balloons are given out. In this situation, division is used to describe *sharing.* A more formal term for this kind of problem is *partitive* division—a division situation in which something is distributed, and the problem is to determine *how many are in each group.*

In the second situation, the action to solve the problem is making groups—that is, making a group of two, then another group of two, and another, and so on until no balloons are left. In this situation, division is used to describe *grouping.* This situation is sometimes called "measurement division" because the total amount is measured out into equal groups. The formal term for this kind of division situation is *quotative* division—a situation in which *how many equal groups* must be determined.

By working with a variety of problems in this unit, students learn to recognize both of these actions as division situations and to develop an understanding that both can be written in the same way: $18 \div 2 = 9$. Depending on the context, help students interpret the notation as either "Divide 18 into two groups. How many are in each group?" or "How many 2s are in 18?"

As students become more flexible with division, they will understand that they can solve a sharing problem by thinking of it as grouping or a grouping problem by thinking of it as sharing in order to make it easier to solve.

How many people are on each team if I make 25 equal teams from 100 people?

To solve this problem, it is easy to think, "How many groups of 25 are in 100?" even though the problem is not about groups of 25, but about 25 groups. The numerical answer to this grouping question is also the numerical answer to the sharing problem. Some of your students may soon have an intuitive understanding that they can solve a division problem by thinking about it either way. Students can draw on what they know about multiplication, that 4 teams of 25 is the same number of people as 25 teams of 4 ($4 \times 25 = 25 \times 4$). This understanding is based on the commutative property of multiplication (See **Algebra Connections in This Unit,** page 16.).