Subtraction Strategies

Students' strategies for subtraction fall into three basic categories: 1) subtracting in parts, 2) adding up or subtracting back, and 3) changing the numbers to numbers that are easier to subtract. In this Grade 3 unit, students are primarily focusing on strategies that fall in the first two categories. The emphasis is on ways to break numbers apart into smaller pieces that can easily be either subtracted or added, and knowing how to combine those pieces to get an accurate solution. In order to use these strategies, students must understand the meaning of subtraction and have a good mental model of what is happening in the problem. They must be able to look at the problem as a whole, think about the relationships of the numbers in the problem, and choose an approach they can carry out easily and accurately.

Here are examples of students' strategies using the following problem as an example:

251 − 187 =

Subtracting in Parts

$$\begin{aligned} 251 - 187 &= \\ 251 - 100 &= 151 \\ 151 - \ 80 &= \ 71 \\ 71 - \ \ 7 &= 64 \end{aligned}$$

$$\begin{array}{r} 251 \\ -\ 100 \\ \hline 151 \\ -\ 50 \\ \hline 101 \\ -\ 30 \\ \hline 71 \\ -\ 7 \\ \hline 64 \end{array}$$

These two students subtracted 187 in parts. The first student broke up 187 by place (100, 80, 7), while the second student subtracted 100 first, and then broke the 87 into numbers (50, 30, 7) easier to work with. When students use this strategy, they should be encouraged to subtract the largest parts they can use, while still making sense of the problem and the numbers.

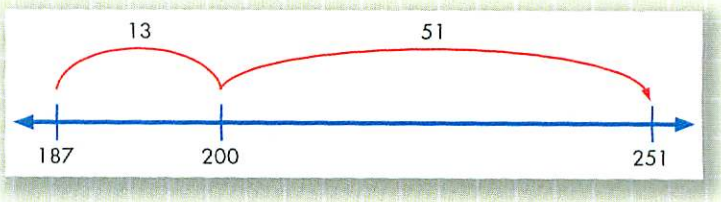
Adding Up or Subtracting Back

In this category of strategies, students visualize how much more or less one number is than the other, and either "add up" or "subtract back" to find their answer. They often represent the subtraction as the distance between two numbers on a number line. In Set A, students start at 187, and "add up" until they reach 251.

Set A: Adding up

$$\begin{aligned} 251 - 187 &= 200 \\ 187 + \underline{13} &= 200 \\ 200 + \underline{51} &= 251 \\ 13 + 51 &= 64 \end{aligned}$$

$$\begin{aligned} 187 + \underline{20} &= 207 \\ 207 + \underline{40} &= 247 \\ 247 + \underline{4} &= 251 \\ 20 + 40 + 4 &= 64 \end{aligned}$$



Both students thought of the solution as how much more must be added to 187 to get a sum of 251. Implicitly, they are using the inverse relationship of addition and subtraction to solve the problem. As shown on the number line, the first student added 13 to 187 to get to 200, then added 51 to get to 251. The second student added multiples of 10 to get very close to 251, then added on the final 4.

Set B: Subtracting Back

In this set of solutions, students started at 251, and then "subtracted back" until they reached 187.

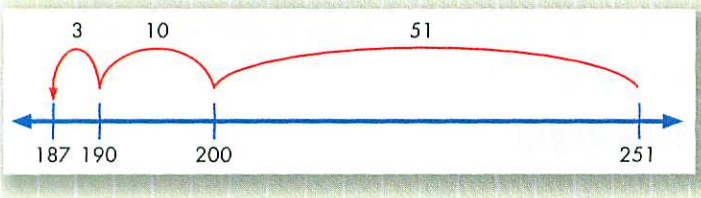
$251 - \underline{51} = 200$

$200 - \underline{13} = 187$

$51 + 13 = 64$

$$\begin{array}{r} 251 \\ -\ 51 \\ \hline 200 \\ -\ 10 \\ \hline 190 \\ -\ 3 \\ \hline 187 \end{array}$$

$51 + 10 + 3 = 64$



Both students solved the problem by "going back" to 200 and then "back 13 more" to 187. As you can see, in the number line representations, students are not subtracting 187 in parts as in the first category; rather, they start at 251 and subtract until they reach 187, then determine how much they subtracted.

Changing the Numbers

In this category of strategies, students change one or both of the numbers to what they often call "landmark" or "friendly" numbers. In the following examples, students changed one or both of the numbers, subtracted, then compensated for the changes they had made.

Changing and Compensating

$251 - 187 =$

$251 - 200 = 51$

$51 + 13 = 64$

$250 - 200 = 50$

$50 + 14 = 64$

The first student changed 187 to 200 to create an easier subtraction problem. Since the student had subtracted 13 too much, 13 was then added to 51 to get the final answer. The second student changed both numbers and had to decide how both those changes affected the result. Since the difference between the two numbers was decreased by 1 (changing 251 to 250) and by 13 (changing 187 to 200), 14 is added to 50. Visualizing the effect of the changes and, therefore, how to compensate for those changes, is critical to this kind of strategy. Changing both numbers as in this second strategy is not easily understood by most third graders. However, changing one number is a strategy that some third graders can use effectively. Number lines are particularly useful tools for visualizing how changing numbers affects the result.

Another strategy in this category is to change both numbers in order to create an equivalent problem that can then be solved without any need to compensate for changes. For example, rather than adding 13 at the end in the first solution above, 13 is added both to 187 and to 251, which maintains the difference between the two numbers ($451 - 187 = 464 - 200$). Students will have an opportunity to explore this strategy further in later grades.

At the end of Grade 3, students should feel comfortable and confident with at least one strategy for subtraction with 3-digit numbers, and be using it with some efficiency—working with larger parts of the numbers and combining steps in their solutions.