Teacher Note

Adding and Subtracting Fractions

The work in this unit on fraction addition and subtraction focuses on using representations and on students' growing knowledge of fraction equivalents to solve problems with familiar fractions. In many cases, the fractions have related denominators (e.g., one denominator is a multiple of the other, such as $\frac{1}{6}$ and $\frac{7}{12}$). However, students also can add some fractions with unrelated denominators (e.g., $\frac{1}{4} + \frac{1}{3}$,) by using other equivalents they know. There are two basic strategies students are developing during this unit. These are illustrated by using Problem 1 on *Student Activity Book* page 46: $\frac{1}{4} + \frac{2}{3}$.

Using a Representation of Fractional Parts

Students may use a rectangle or rotation on a clock to solve the problem $\frac{1}{4} + \frac{2}{3}$.

Rectangle representation

When using rectangles to solve an addition problem, students need to choose a rectangle with an area that is easily split into the number of parts represented by each denominator. In this case, either the 4 × 6 or the 5 × 12 rectangle could be used. (A problem involving fifths and fourths would lend itself to use of the 5 × 12 rectangle, but not the 4 × 6 rectangle.) A student's solution using the 5 × 12 rectangle might be:

Students might say:

"I know that the whole rectangle is 60 square units. $\frac{1}{2}$ of that is 30, so $\frac{1}{4}$ is 15. If you split 60 into 3, that's 20, so $\frac{2}{3}$ is 40. When I add them up, that's 55 square units altogether. So then I said the answer was $\frac{55}{60}$. Ms. R. said that was right, but that there was another fraction that was simpler and that I should think about 5s. So then I finally realized that every 5 is $\frac{1}{12}$, and so it's $\frac{11}{12}$."

As this student points out, using these representations can result in an unfamiliar fraction, a fraction that is not yet in lowest terms. However, what this student has done is at the core of understanding addition: the student has represented both fractions correctly, combined them, and identified the fraction that represents the sum. Figuring out that $\frac{55}{60} = \frac{11}{12}$, as the teacher urged, helps the student use the representation more fluently—recognizing that in this representation every five square units represents $\frac{1}{12}$ of the whole area.

Clock representation

Other students might choose the clock representation to solve this problem, since the denominators lend themselves to parts of the rotation around the clock. This student thought about the rotation of the hour hand.

Students might say:

"One fourth of the way around is three hours. One third around is four hours, because it's $\frac{1}{3}$ of 12, that's four. So $\frac{2}{3}$ is eight hours. That's three plus eight is 11 hours. The answer is $\frac{11}{12}$."

If a student thinks about minutes (15 minutes + 40 minutes = 55 minutes), the student may come up with an answer of $\frac{55}{60}$. Here, too, encourage such students to find a more familiar fraction, perhaps, by asking, "What if you thought about the hour hand instead of the minute hand?"

Students who use representations such as these have to keep straight how their representation represents one unit whole. For example, a student who used the clock may at first say that the answer to this problem is "11 hours." You may ask, "So if I write $\frac{1}{4} + \frac{2}{3} = 11$, is that what you mean?"

Using Fraction Equivalents

Throughout their fraction work in Grades 3–5, students gradually learn more and more fraction equivalents, which they can use to solve problems. From their work with fractions on the clock, many students may recognize that both $\frac{1}{4}$ and $\frac{2}{3}$ have fraction equivalents with a denominator of 12.

Students might say:

"I know that $\frac{1}{4} = \frac{3}{12}$, because on the clock $\frac{1}{4}$ way around is like three hours out of 12. And $\frac{1}{3} = \frac{4}{12}$, so $\frac{2}{3} = \frac{8}{12}$. I can add $\frac{3}{12}$ plus $\frac{8}{12}$: $\frac{3}{12} + \frac{8}{12} = \frac{11}{12}$."

This student is, in fact, finding common denominators. By visualizing how $\frac{3}{12}$ is three out of 12 equal parts and $\frac{8}{12}$ is eight out of 12 equal parts, students come to know that they can combine fractions with the same denominator simply by adding the numerators.

Although subtraction of fractions is not a benchmark for this unit, students do solve subtraction problems throughout the unit. They base their computation on their understanding of subtraction and addition as having an inverse relationship, which they have developed through their work with whole numbers. Students approach subtraction of fractions in the same way that they solve addition problems, through using fraction equivalencies and representations. They discuss this relationship and how it applies to fractions in Session 3.3 and Session 3.7.

In later grades, students will encounter algorithms for computing with fractions. For example, they will learn about using common denominators to solve addition problems. To add $\frac{7}{12}$ and $\frac{29}{72}$, for example, common denominators are very useful. Students would not be expected to be able to add these fractions after completing this unit, but they should be able to estimate that they are adding a fraction that is a little more than $\frac{1}{2}$ and a fraction that is a little less than $\frac{1}{2}$, so the sum would be close to 1. They are also learning how common denominators are useful for computing with fractions, given their expanding knowledge of fraction equivalents, as in the example of adding $\frac{1}{4}$ and $\frac{2}{3}$ above. In this unit, the focus is on developing meaning for fractions and percents. The work on addition and subtraction in this unit is a context for students to use this understanding and to expand their knowledge of fraction relationships.