Comparing Fractions to Landmarks

This Grade 4 class is working on the activity Comparing Fractions to Landmarks in Session 2.4. They are sorting their Fraction Cards and placing them on a number line in relationship to the landmark numbers 0, $\frac{1}{2}$, 1, and 2. The teacher listens and questions a group about how they are making their decisions as they work.

Ramona: I'm going to put $\frac{1}{3}$ between 0 and $\frac{1}{2}$. Does everyone agree?

Jill: I do. It's definitely not more than $\frac{1}{2}$.

Teacher: Why do you say that, Jill?

Jill: I know that $\frac{1}{3}$ of a rectangle is less than one half of the rectangle because when you break a rectangle into three pieces instead of two, the pieces of three are smaller than the pieces of two.

Teacher: What if the fraction was $\frac{2}{3}$? Where would that go?

Bill: That would go after the $\frac{1}{2}$ because $\frac{1}{3}$ is less than $\frac{1}{2}$ but $\frac{2}{3}$ is more than $\frac{1}{2}$.

Teacher: But how do you know that? I know that you can see it on your cards, but can anyone explain it—as if you were explaining it to a third grader?

Yuki: It is bigger than $\frac{1}{2}$ but smaller than 1. It is definitely smaller than 1 because $\frac{3}{3}$ would be 1 whole and $\frac{2}{3}$ is less.

Bill: And you know it has to be more than $\frac{1}{2}$ because look, there are three pieces if it's thirds, and so it takes one and a half pieces to make one half, but $\frac{2}{3}$ is two of the pieces, so it's more.

Ramona: Uh oh, look at this one. $\frac{5}{3}$. Where does this one go?

Yuki: It goes between 1 and 2. I think it's larger than 1 because 1 is $\frac{3}{3}$ and $\frac{5}{3}$ is $\frac{2}{3}$ more than $\frac{3}{3}$, so $\frac{5}{3}$ is bigger than 1.

Jill: I agree. If the top number is bigger than the bottom number it is bigger than 1 whole. It's one of the conjectures on our list.

Bill: But how do you know that it is?

Teacher: This is what you were saying you didn't get the other day. Can you say again what you were saying about a fraction like $\frac{5}{3}$?

Bill: I get it a little better now, but I was saying how can you have five thirds when you can only have three thirds?

Teacher: So what do you mean, "you get it a little better now"?

Bill: When Tonya was saying yesterday that $\frac{5}{3}$ is just the same as 1 and $\frac{2}{3}$. That makes more sense to me. If it's a sandwich, it's one whole sandwich and $\frac{2}{3}$ of another one.

Ramona: It's just another way to say it. Three thirds and two thirds is five thirds, or it's one and two thirds.

Bill: I still think that five thirds sounds weird, though.

Teacher: Yuki used what he knew about 1 to help him decide where $\frac{5}{3}$ goes. If we added $1\frac{1}{2}$ to our landmark fractions, where would you put $\frac{5}{3}$? Is it bigger or smaller than $1\frac{1}{2}$?

Yuki: Like Bill and I said, $\frac{2}{3}$ is bigger than $\frac{1}{2}$, so $1\frac{2}{3}$ is also bigger than $1\frac{1}{2}$.

Bill: The next card is $\frac{2}{6}$. Where would $\frac{2}{6}$ go?

Ramona: On top of $\frac{1}{3}$ because $\frac{2}{6}$ is equal to $\frac{1}{3}$.

Teacher: How do you know that they're equal?

Jill: When I look at the part that's colored in my Fraction Card, they're the same.

Teacher: Again, could you explain it more? If you weren't looking right at the cards, how could you convince me?

Jill: When you cut thirds in half, you get sixths. First you have three pieces, and then you have six pieces. Each of the thirds makes two sixths when you cut it in half.

In this discussion, the teacher encourages students to explain their reasoning to one another and to extend their thinking (is $\frac{5}{3}$ more or less than $1\frac{1}{2}$?). The teacher wants her students to use the pictures on their Fraction Cards to help them visualize relationships between fractions and landmarks such as $\frac{1}{2}$ and 1, but she also wants them to use what they know about fractions to reason about the relative size of the fractions they are placing on their number line. When Jill says, "I look at the part that's colored in my Fraction Card," the teacher asks for further explanation.

Similarly, when students simply assert that one fraction is smaller than another, as when Bill compares $\frac{1}{2}$ and $\frac{2}{3}$, the teacher asks further questions. The teacher also pursues Bill's comment about $\frac{5}{3}$ because a number of students are not clear about the meaning of fractions greater than 1. Although the teacher is fairly confident that Jill can articulate good reasoning to back up her assertion that "If the top number is bigger than the bottom number, it is bigger than one whole," rules like this are sometimes readily adopted by other students without the understanding that Jill has, so the teacher wants to get a sense of how others in the group are interpreting the meaning of $\frac{5}{3}$.