

Does Order Matter in Addition?

In third grade, the question of order in addition can come up for students in a variety of situations.

- When representing addition problems with stickers and strips: “ $26 + 52$ and $52 + 26$ are both 7 strips and 8 singles.”
- When working on the activity *How Many More Stickers to Get 100?* “If I start with 48, I need 52. If I start with 52, I need 48.”
- When adding more than 2 numbers, for example, $34 + 23 + 16$: “I did it this way: $34 + 16$ is 50, and $50 + 23 = 73$.”

Many students already realize that when the order of the addends in an addition problem changes, the sum does not. They may explain that this happens because you are just changing the placement of the numbers, but “you are not adding any more or taking anything away.”

From a formal mathematical perspective, the ideas these students are working on involve two basic laws of arithmetic. The first is the *commutative law of addition*, which states that two numbers added in either order yield the same sum. Some students call these “opposites,” or “switch-arounds,” or “reversibles.” The very fact that students have given names to this phenomenon indicates that they have formulated a generalization: If you take two numbers and switch them around, you still get the same sum when you add them. Written algebraically, this law can be expressed as $a + b = b + a$.

The second basic law, the *associative law of addition*, states that when three numbers are added together, they can be regrouped without changing the order and will yield the same sum. For example, $(35 + 14) + 6 = 35 + (14 + 6)$. In the expression to the left, first add $35 + 14$ to get 49, and then

add $49 + 6$. In the expression to the right, first add $14 + 6$ to get 20, and then add $35 + 20$. (Using parentheses indicates that the operation within the parentheses is to be carried out first.) The sum for both is 55. Written algebraically, this law can be expressed as $(a + b) + c = a + (b + c)$. One calculation is easier than the other; the associative property guarantees that the sum is constant.

When performing addition with two numbers, the commutative law applies. When adding three or more numbers, the reordering might involve the commutative law, the associative law, or both in combination. It is not important for students to learn the formal names of these properties. Rather, they should be encouraged to examine questions about order, and to support their reasoning with various ways to represent addition—such as combining stickers and strips, story contexts, drawings of the situations, number lines, or 100 charts.

As students continue to learn about operations and calculations, questions about order will repeatedly arise: Does order matter when you subtract? What about when you multiply? What about when you divide? (They will find that it does matter when subtracting or dividing but not when adding or multiplying.) Does it matter when you add fractions or integers (which include numbers below zero)? Answering these questions is work ahead of your students in the months and years to come.