Conjectures About Fractions

As students work on their Fraction Number Lines, the class has been listing conjectures about comparing fractions. In the discussion at the end of Session 2.6, the teacher focuses on two of the ideas many of the students have been using.

Teacher: You've come up with quite a few conjectures about rules for comparing fractions. One that I noticed many of you have been using is this one [points to the chart]: *If the denominators of two fractions are the same, the one with the larger numerator is larger.* Benson, weren't you using this idea when you were trying to decide where to put $\frac{6}{8}$ on the number line?

Benson: $\frac{6}{8}$ would go in here [points to a spot midway between $\frac{1}{2}$ and 1 on the class Fraction Number Line] because $\frac{4}{8}$ would be equivalent to $\frac{1}{2}$. So, because 6 is greater than 4, it would be more pieces of 8, so $\frac{6}{8}$ would be bigger than $\frac{1}{2}$. It wouldn't be $\frac{8}{8}$, so it would be between $\frac{1}{2}$ and 1.

Teacher: Does this idea work with any pair of fractions? If the denominators are the same, but the numerators are different, the one with the larger numerator is a larger fraction? What is another pair of fractions like this?

Sabrina: $\frac{2}{4}$ and $\frac{3}{4}$. That's an easy one. $\frac{3}{4}$ is larger.

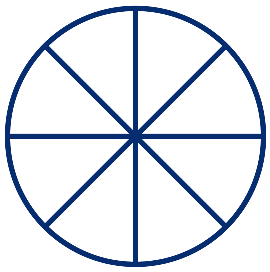
Andrew: $\frac{5}{12}$ and $\frac{11}{12}$. $\frac{11}{12}$ has to be bigger because it's 11 of the twelfths.

Teacher: Benson said that it would be "more pieces of eight" and Andrew said that "it's 11 of the twelfths." Do you think this idea works with any pair of fractions with the same denominators? Do you think it's always true?

Marisol: It has to be because you're breaking something into the same number of pieces but you're taking more of them.

Anna: It's like if you had pizza. You cut it into 8 slices. One slice is $\frac{1}{8}$. If I eat $\frac{3}{8}$ and you eat $\frac{5}{8}$, you ate 5 slices. You ate more.

Teacher: Here's what I think Marisol and Anna are saying [quickly sketches a circle divided into eight equal pieces]. They're saying that $\frac{3}{8}$ is three of the 8 slices, and $\frac{5}{8}$ is five of the 8 slices, so it's more slices. Can anyone use this pizza story and picture to talk about why this would be true for any pair of fractions with the same denominators? Would it work for $\frac{5}{19}$ and $\frac{8}{19}$ or $\frac{7}{10}$ and $\frac{9}{10}$?



Jake: It has to work. It doesn't matter how many pieces you make. Whatever the number of pieces, that's the denominator. So if you take more of the pieces, you are taking more of the whole pizza.

Lucy: You could have 100 pieces, and still if one person has $\frac{3}{100}$ and one person has $\frac{19}{100}$, the person with the higher numerator has more. The higher number is more of the same thing.

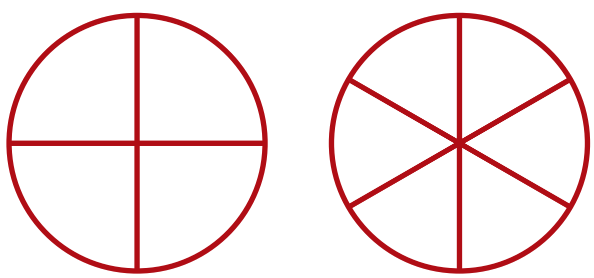
Teacher: You have some good arguments for comparing fractions with the same denominator. Let's look at one of your other conjectures, which is about fractions that have the same numerators. The conjecture we wrote down is this: *If two fractions have the same numerator, the one with the larger denominator is a smaller fraction.* First of all, what are some examples of fractions that have the same numerator but different denominators?

After listing some examples, the discussion continues.

Ursula: Ummm . . . it's like when I was putting $\frac{2}{6}$ on the number line, and I didn't know whether it went before $\frac{1}{4}$ or after it. So I knew $\frac{2}{6}$ is equal to $\frac{1}{3}$ and they both have 1 at the top. 3 is smaller than 4, so it's bigger. $\frac{2}{6}$, which is equal to $\frac{1}{3}$, comes after $\frac{1}{4}$.

Teacher: You're saying that $\frac{1}{4}$ and $\frac{1}{3}$ have the same numerator, but 3 is smaller than 4, so $\frac{1}{3}$ is larger. But how do you know that works for these two fractions or for other pairs of fractions like this?

Venetta: You can do it the same way, with pizzas. I could eat $\frac{1}{4}$ of a pizza and you could eat $\frac{1}{6}$ of a pizza. I'd eat more.



Teacher: Is there a way I could draw that?

Venetta: Draw one with four slices and one with six slices. The one with four slices has bigger slices, so one of the four slices is more than one of the six slices.

Teacher: Who can say anything else about this?

Enrique: It's like when I was doing $\frac{5}{3}$ and $\frac{5}{4}$. I didn't know which one came first, but then I thought about thirds and fourths. Thirds are bigger parts, so each of the thirds is bigger than each of the fourths, so the whole thing is bigger.

The teacher helps students start with particular examples and then use those examples to state more general ideas about the fraction relationships. Lucy says, "The higher number is more of the same thing." Jake says, "Whatever the number of pieces, that's the denominator. So if you take more of the pieces, you are taking more of the whole pizza." In these statements, Lucy and Jake are making general claims about all pairs of fractions with the same denominator. As students talk, the teacher uses familiar fraction representations so that all students can visualize the relationships to which students refer in their arguments and have the chance to build on those ideas themselves.