Multiplying with Fractions

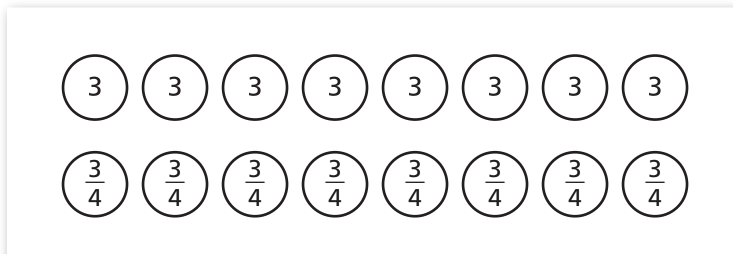
When students learn to add and subtract fractions, they extend the images they have developed about addition and subtraction of whole numbers to their work with fractions. Considering addition as joining two quantities and subtraction as removal, comparison, or finding the difference still make sense to students as they work with fractions. For example, in Unit 3, students solved problems like these:

- The Smith family had a pan of cornbread. One night they ate $\frac{3}{8}$ of the pan, and another night they ate $\frac{7}{12}$ of the pan. What fraction of the pan of cornbread did they eat altogether?
- There was a giant brownie on a plate at a birthday party. Ben ate $\frac{5}{12}$ of the brownie. Walter ate $\frac{2}{5}$ of the brownie. How much more of the brownie did Ben eat?
- Lourdes was riding her bike on a bike path that was $6\frac{2}{3}$ miles long. She rode her bike $4\frac{7}{9}$ miles and stopped to rest. How much farther does she need to ride until she's at the end of the bike path?

The first problem is about joining two quantities, the second is about comparing two quantities, and the third is about finding the difference between two quantities. Solving these problems may be challenging as students think through how to compute with the quantities in the problem, but it does not require students to rethink their fundamental ideas about how the operations of addition and subtraction work.

When students begin to multiply and divide with fractional quantities, they find it more difficult to generalize the images they have used with whole numbers. Furthermore, the relationship of the size of the factors to the size of the product or the size of the dividend to the size of the quotient is not the same as what students have come to expect in working with whole numbers.

Students' understanding of multiplication as involving groups of equal groups is a useful starting point. In Grade 4, students started their work with multiplying with fractions by using contexts like traveling the same distance each day (Ms. Clark travels to work $\frac{3}{4}$ mile each day), using the same quantity repeatedly (a rabbit eats $\frac{3}{8}$ cup of carrots every day), or buying fabric (each piece of fabric is $\frac{7}{8}$ yard long). They solved problems such as "How many miles does Ms. Clark travel in 8 days?" or "If I need 7 pieces of fabric, how many yards will I have to buy?" In all of these contexts, a fractional amount is used repeatedly, so the fractional amount is multiplied by a whole number (e.g., 8 days $\times \frac{3}{4}$ mile per day, or $8 \times \frac{3}{4}$). This type of problem, which students review at the beginning of this unit, can be understood in the ways that students have been thinking about multiplication of whole numbers as groups of groups. That is, $8 \times \frac{3}{4}$ can be pictured in a way similar to 8×3:



Students are used to the language of "number of groups" and "number in each group" when working with whole numbers, and while it requires a transition to think of the "number in each group" as a fractional amount, students are generally able to make this shift within their model of multiplication. They see why 8 groups with $\frac{3}{4}$ in each group would be modeled as $8 \times \frac{3}{4}$, because they are interpreting the multiplication symbol as meaning "groups of."

However, in this unit, when students start to work with contexts and problems that deal with a fractional amount of a whole number (and, later, a fraction of a fractional amount), it is more difficult to use their previous images of multiplication to make sense of these problems. In order to understand this better, consider this problem:

> Ms. Clark drives 8 miles to work each day. One day, her car breaks down when she is $\frac{3}{4}$ of the way there. How far did she drive before her car broke down?

In the problem previously discussed, 8 days $\times$ $\frac{3}{4}$ mile, $\frac{3}{4}$ represented a measurable quantity that could be iterated 8 times. In this second type of problem, the number $\frac{3}{4}$ can only be understood in relation to the whole distance of 8 miles. Students often solve this type of problem by dividing 8 miles into fourths and taking three of those fourths in order to come up with the correct answer of 6 miles. See **Teacher Note 2:** Strategies for Solving Multiplication and Division Problems with Fractions for examples of how students might represent this problem. However, they do not necessarily *see how this problem is multiplication* and do not know how to write a multiplication equation to model the situation mathematically. In fact, many students at first think that what they have done should be modeled with division because they *divided* 8 miles into fourths.

Some students write equations using whole numbers: $8 \div 4 = 2$, $3 \times 2 = 6$. While these equations do correspond to the actions students have taken in making their representations, they do not involve the numbers in the original problem. That is, these equations do not show how $\frac{3}{4}$ and 8 are related to the answer of 6. Other students write $8 \div \frac{1}{4} = 2$ for their first step, again followed by $3 \times 2 = 6$, incorrectly thinking that the first equation means "8 divided into fourths equals 2."

While many students actually find the correct answer, they do not see the problem as multiplication. Students' images of multiplication as groups of equal groups do not seem to fit these problems. Through experience and discussion, they need to learn that contexts like these are still about groups, but about a part of one group rather than about putting together several groups. It is often helpful for students to see a series of multiplication problems in the same context, moving from whole numbers to fractions:

Ms. Clark drives 8 miles to work each day.

How many miles does she drive in 5 trips?	5 groups of 8	$5 \times 8 = 40$
How many miles does she drive in 2 trips?	2 groups of 8	$2 \times 8 = 16$
How many miles does she drive in 1 trip?	1 group of 8	$1 \times 8 = 8$
How many miles does she drive in $\frac{3}{4}$ of a trip?	$\frac{3}{4}$ of a group of 8	$\frac{3}{4} \times 8 = 6$

Rather than expressing a multiple of 8, $\frac{3}{4} \times 8$ represents a part of a group of 8. Such a sequence can help students learn why it makes sense to interpret $\frac{3}{4}$ of 8 as multiplication.

While some time is given to explicitly discuss with the whole class as to why the problem situations they encounter in this unit are multiplication, keep in mind that many students will need to keep thinking about this as they work on different problem sets. As you interact with individual students while they are solving problems, take those opportunities to continue the conversation about why a particular problem is multiplication.