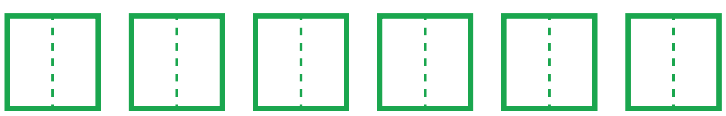
Dividing a Whole Number by a Fraction and a Fraction by a Whole Number

In this unit, students are introduced to division with fractions. The numbers in the problems students solve are small and the fractions are all unit fractions, so students can focus on the conceptual issues that involve division with fractions. What is difficult for students to understand is what the problems mean, how the quantities in the problem are related, what whole a fraction is part of, and how a division equation corresponds to the problem context. See **Teacher Note 4:** Dividing with Fractions for a further discussion of what coming to understand division with fractions entails.

Students generally sketch their own representations for these problems using a series of simple objects if they are splitting up a whole number or using a single rectangle if they are splitting a single whole into fractional parts. For example, to represent $6 \div \frac{1}{2}$, students might sketch the following.



$6 \div \frac{1}{2} = 12$

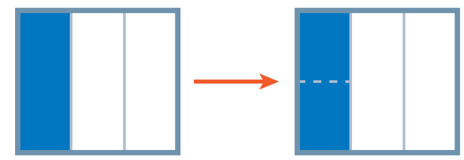
Rachel: I thought of it as "How many halves are in 6?" So I drew 6 boxes and divided them all in half. There are 12 halves.

Once students understand how to interpret an expression such as $6 \div \frac{1}{2}$, some students will reason about the numbers without drawing a representation.

Terrence: I know that there are two halves in every whole, so 2×6 is 12.

Note that this solution, which makes complete sense, also raises the issue for students about whether this problem is a multiplication or division problem. The equation $2 \times 6 = 12$ represents one of the steps Terrence used to solve the problem, but it does not represent the original problem and its solution. Students are, at first, surprised that the answer to the original division problem can be greater than either of the numbers in the problem. They discuss how the problem is division and why the size of the answer makes sense in Session 1.10.

Dividing a fraction by a whole number, for example, $\frac{1}{3} \div 2$, looks even stranger to students. Before they can represent this problem, they need to learn to interpret what the division is—dividing $\frac{1}{3}$ into 2 parts. See **Teacher Note 4:** Dividing with Fractions. A rectangle works well to represent this type of problem. First students represent $\frac{1}{3}$ of a whole, and then divide that $\frac{1}{3}$ into two equal parts.



Once students have learned to interpret an expression like this one, some students reason about the number relationships to solve the problem.

Hana: Half of a third is a sixth, so if you split $\frac{1}{3}$ into two parts, you get $\frac{1}{6}$.

Using the Properties and Relationships of the Operations

As students develop images of multiplying and dividing with fractions, they gradually expand their notions of multiplication and division to include these numbers and come to realize that the properties, behaviors, and relationships of these operations still hold. For example, multiplication is commutative. Students are familiar with this property of multiplication with whole numbers, but they may not at first be convinced that $\frac{1}{4} \times 280$ and $280 \times \frac{1}{4}$ are equal to the same product. While students are likely to use different representations and story contexts for these two expressions, eventually they will realize that they can solve multiplication problems with fractions in any order, regardless of the context, just as they can with whole numbers. If students in your class bring up the question of whether they can "turn the numbers around" in a multiplication problem involving fractions, ask them to investigate different examples and begin to think about whether this property holds with fractions.