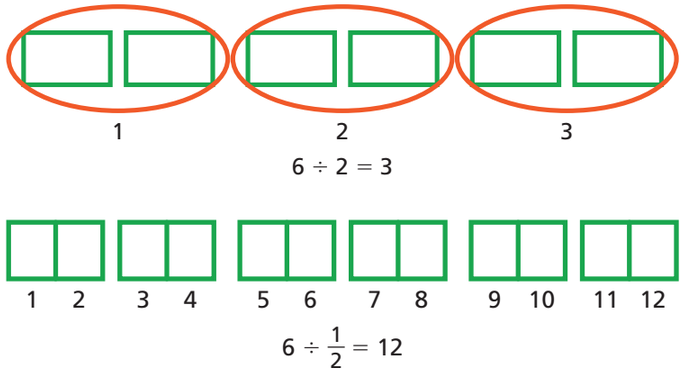
Dividing with Fractions

Students encounter division with fractions for the first time in this unit. As in multiplication, they should draw on what they know about the meaning of division with whole numbers, but will also need to expand their understanding of division to accommodate some of the problems they encounter in this unit. Students should have two interpretations of division from their work with whole numbers to draw on. When they solve a problem such as $6 \div 3$, they might use either of these interpretations: 1) "How many 3s are in 6?" or 2) "If I divide 6 into 3 parts, how big is each part?" Students should be able to use both of these interpretations of division and choose which one serves them best for particular problems. For example, to solve $240 \div 3$, one might choose to think of dividing 240 into 3 parts, but for $240 \div 80$, one might choose to think about how many 80s are in 240.

The first type of division problem that students work with in this unit is a whole number divided by a unit fraction, for example, $6 \div \frac{1}{2}$. Students need to learn how to read a problem like this so that it matches one of the two interpretations of division. Thinking of this problem as "How many $\frac{1}{2}$s are in 6?" helps students see why their quotient ends up larger than either 6 or $\frac{1}{2}$. However, some students at first try to apply their other image of division, dividing the dividend into parts. It is difficult to make sense of reading this problem as "6 divided into $\frac{1}{2}$ of a part," so students who think about dividing into parts often misread this problem as "6 divided in half," and end up with an incorrect quotient of 3. They are interpreting $6 \div \frac{1}{2}$ as if it were $6 \div 2$. One question to ask students is, "Could $6 \div 2$ and $6 \div \frac{1}{2}$ have the same answer?" This question will help them start to think about how these two expressions are different. Then they can use representations and story contexts to help them visualize what the quantities 6 and $\frac{1}{2}$ mean and how they are related by division. For example, here is a story context that students might use to compare $6 \div 2$ and $6 \div \frac{1}{2}$.

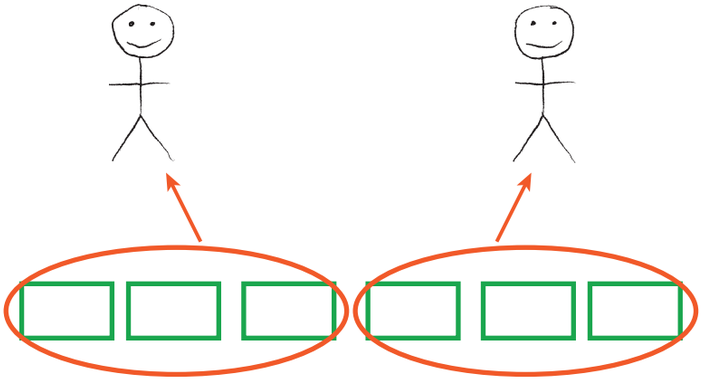
- Talisha has 6 cups of flour. She needs 2 cups for each batch of cookies. How many batches can she make? ($6 \div 2 = 3$)
- Talisha has 6 cups of flour. She needs $\frac{1}{2}$ cup for each batch of cookies. How many batches can she make? $\left(6 \div \frac{1}{2} = 12\right)$



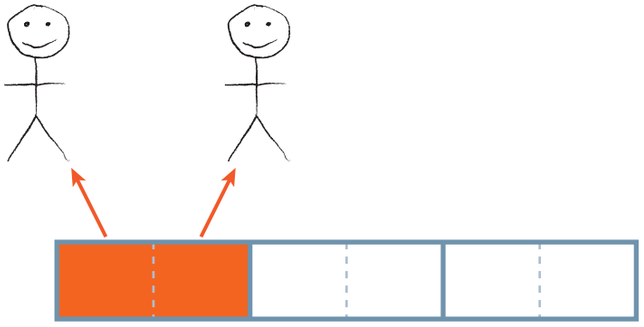
Just as in multiplication, an important part of the work for students is understanding why the second problem is division and how the different numbers in the written equation correspond to the different quantities in the context. As they work with these problems, they are expanding their ideas about whether the answer to a division problem is greater or smaller than the number being divided (i.e., comparing the quotient to the dividend). For most whole-number division problems they have encountered, the dividend was greater than the divisor, so that, as long as the divisor was greater than 1, the quotient was always smaller than the dividend. Through using contexts and representations, students are learning that the quotient can also be greater than the dividend.

The second type of division problem in this unit is a fraction divided by a whole number, for example, $\frac{1}{3} \div 2$. For these problems, students' common interpretation of division as taking groups of the divisor out of the dividend is not going to work; it does not make sense to read this problem as "How many 2s are in $\frac{1}{3}$?" (It actually is possible to interpret the problem in this way. The answer to "how many 2s are in $\frac{1}{3}$?" is that there is $\frac{1}{6}$ of a 2 in $\frac{1}{3}$. However, this way of conceptualizing the problem is generally not helpful for Grade 5 students.) Their other interpretation of division works better here: "Divide $\frac{1}{3}$ into 2 parts—how much is in each part?" It is helpful for students to compare this problem to a problem with whole numbers in a context that makes sense for this type of problem.

- Talisha and her friend have 6 cups of flour to use for baking brownies. If they share the flour equally, how much flour does each person get? $(6 \div 2 = 3)$
- Talisha and her friend have $\frac{1}{3}$ of a cup of flour to use for baking brownies. If they share the flour equally, how much flour does each person get? $\left(\frac{1}{3} \div 2 = \frac{1}{6}\right)$



$6 \div 2 = 3$



$\frac{1}{3} \div 2 = \frac{1}{6}$

Throughout this work, help students come up with story contexts and representations that help them visualize the relationships among the quantities in these division and multiplication problems. Connecting the quantities and the operation use in the context to the numbers and operation in the written equation is crucial in order for students to make sense of these problems.