

Approaches to Addition and Subtraction Problems

Students commonly take one of these approaches to solving an addition or subtraction problem:

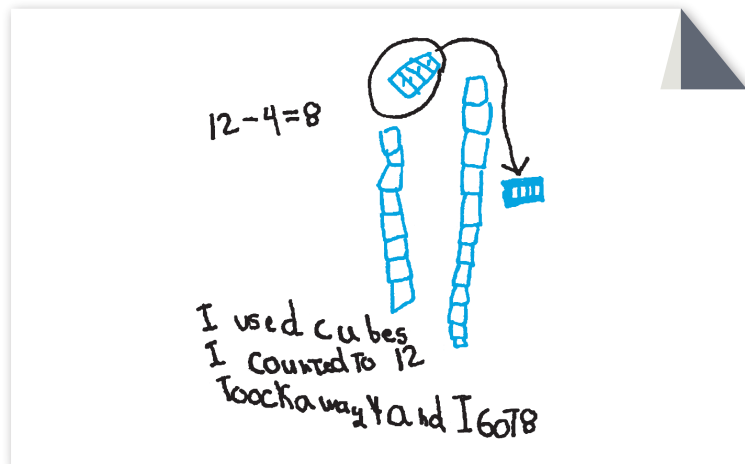
- Counting All
- Counting On/Back
- Counting or Adding Up (subtraction only)
- Numerical Reasoning

Each approach is described below and illustrated with examples of student work on the following problem:

There were 12 squirrels on the ground. Then 4 of them ran up a tree. How many stayed on the ground?

Counting All

When young students first encounter story problem situations, they usually directly model the problem in order to solve it. They draw pictures, or, like Edgar, they count out 12 cubes, take away 4, and count the ones that remain.



[Edgar's Work]

As students gain skill in visualizing addition and subtraction problem situations and begin to develop a repertoire of number relationships they know, they gradually develop other strategies based on counting on or counting back and on numerical reasoning. These strategies require visualizing all of the quantities in the problem and their relationships, and recognizing which are known and which need to be found.

Counting On or Counting Back

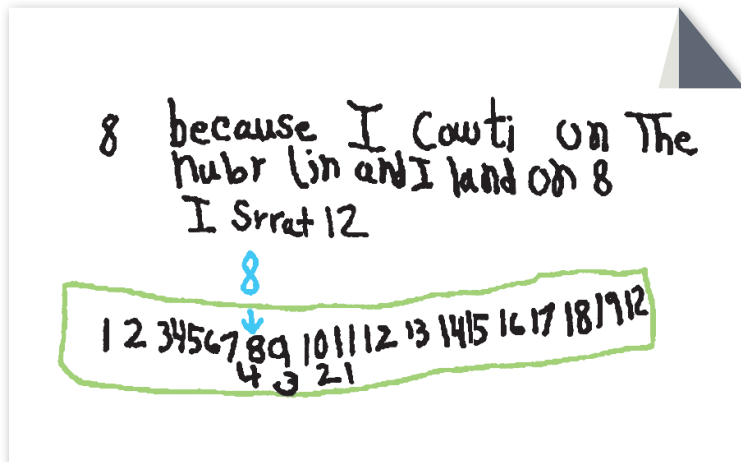
Some students, who perhaps feel more confident visualizing the problem mentally, use strategies that involve counting on or counting back.

In the above story problem, Deshawn used his fingers to count back. To get 12, he explained that he used both hands and visualized 2 "imaginary fingers." He counted back from 12, first counting back 2 in his head, using his imaginary fingers, and then counting back 2 more on his actual fingers to get 8.



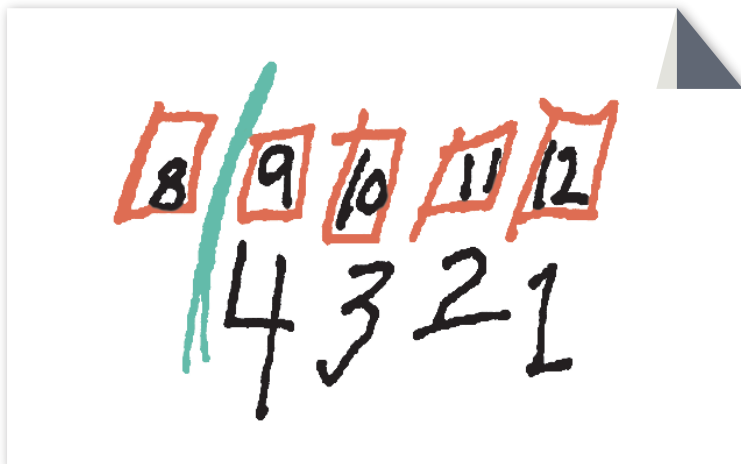
[Deshawn's Work]

Nicky used the class number line. She started at 12 and counted back 4.



[Nicky's Work]

Bruce counted back 4 from 12 in his head.

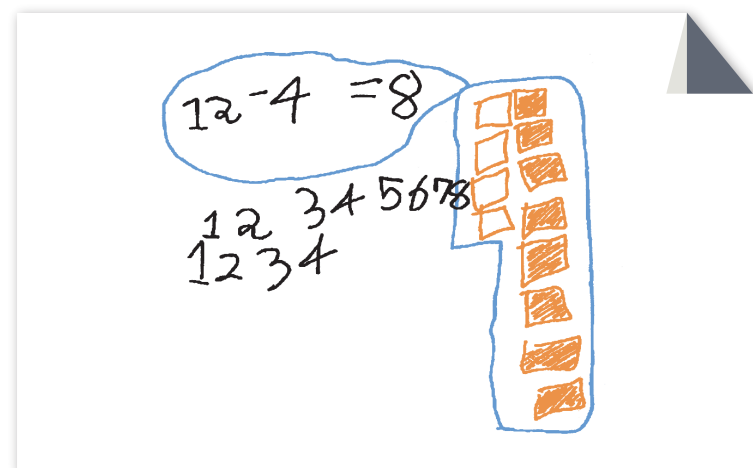


[Bruce's Work]

Although these three students' methods somewhat resemble the methods of the students who directly modeled the action in the problem, there is an important difference: None had to construct the 12 from 1s. Deshawn quickly made the 12 out of larger chunks ($5 + 5 + 2$), and Nicky and Bruce simply started with 12. Counting back for subtraction requires a complex double counting method. These students must simultaneously keep track of the numbers they are counting back (11, 10, 9, 8) and the number of numbers counted (1, 2, 3, 4).

Counting or Adding Up (Subtraction Only)

Instead of counting back 4, Isabel counted up from 4. She counted out 4 cubes in a column to represent the 4 squirrels that ran up a tree. She continued putting cubes in a second column, counting on from 4 until she had a total of 12. Then she counted the number of cubes in the second column. Isabel was able to transform the problem into a missing addend problem: $4 + \underline{\quad} = 12$, counting on to find the solution. See **Teacher Note 8: The Relationship Between Addition and Subtraction** for more information.

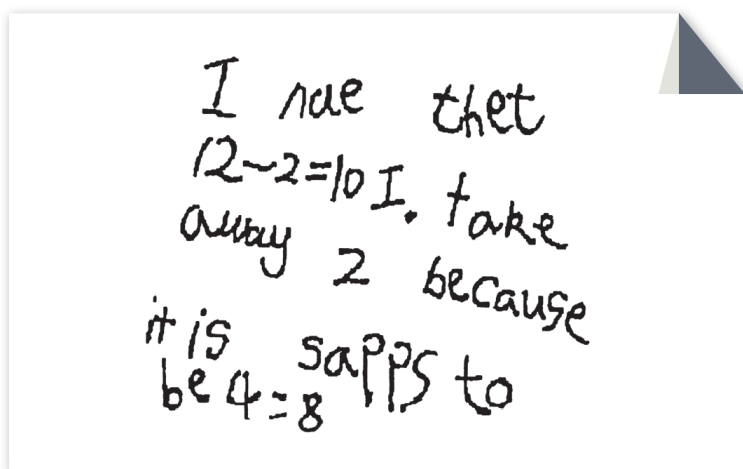


[Isabel's Work]

Numerical Reasoning

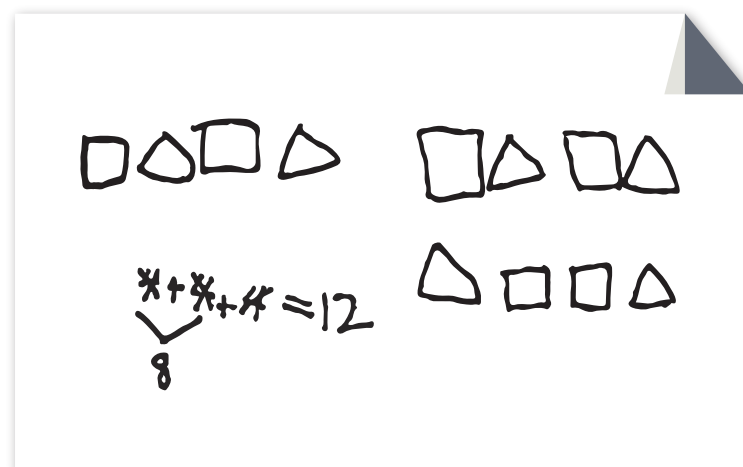
As students learn more about number relationships, they begin to be able to solve problems by taking numbers apart into useful chunks, manipulating those chunks, and then putting them back together.

Lyle broke 4 into 2 and 2. He then subtracted each chunk separately: $12 - 2$ is 10, and then $10 - 2$ is 8.



[Lyle's Work]

Tamika explained, "I know 4 and 4 and 4 is 12. One 4 is the squirrels that ran away. Two 4s is 8, and that's how many are in the tree."



[Tamika's Work]

These students are using strategies that involve chunking numbers in different ways, rather than counting by 1s. They are able to visualize the structure of the problem as a whole in order to identify number relationships they know that might help them solve the problem. It is important to encourage strategies such as these, but keep in mind that ability to work with chunks greater than 1 develops gradually over the early elementary years.