Investigations and the Mathematical Practices

Mathematical Practices in the *Investigations* Curriculum

In the elementary grades, students' mathematical work focuses on four key areas of content—number and operations, geometry, measurement, and data. Just as important, students are learning what it means to do mathematics and they are developing their own mathematical identities. What students learn in these years is critical in terms of how they view mathematics, whether they believe they can have mathematical ideas, whether they are willing to tackle unfamiliar problems, and whether they think of mathematics as intriguing and engaging, or as boring or unapproachable, with rules they do not understand. In these grades, does mathematics invite them in, or shut them out?

Developing an understanding of what it means to do mathematics is fundamentally about the practices of the discipline. *Investigations in Number, Data, and Space* has always integrated into the learning sequence those core mathematical practices that focus on reasoning, communication, and making sense. This third edition of the curriculum makes more explicit the mathematical practices that have always been embedded in the materials. The eight mathematical practices spelled out in the *Common Core State Standards for Mathematics* and the Elementary Elaborations (Illustrative Mathematics, *Standards for Mathematical Practice: Commentary and Elaborations for K–5*) provide a useful and comprehensive framework for highlighting this work. These eight practices are important for students, whether or not their classroom is working with the rest of the *Common Core State Standards*.

What does it mean for mathematical practices to be taken seriously as a part of instruction? Just as students have to learn mathematical content, they also need to learn how to engage in mathematical practices through targeted, intentional, planned instruction. It is not sufficient to post a list of practices on the wall or have students check off when they are engaging in a particular practice. Students must have focused opportunities to learn and practice how to engage in these practices. For this reason, there are two *highlighted practices* in every curriculum unit.

Highlighted Mathematical Practices

Each of the eight practices occurs as a highlighted practice twice during the school year. A *highlighted practice* is chosen for a unit because the particular mathematical content of that unit provides important opportunities for students to learn how to engage in that practice. Practices are always embedded in content. For example, if students are learning to make mathematical arguments (**MP3**), they are making these arguments in the context of the unit's mathematical content. Look for the icon, which indicates the mathematical practice is one of the *highlighted practices* in the unit.

Mathematical Practices in This Unit Essays

This essay, found at the beginning of each unit, describes the two *highlighted practices* and provides examples of how teachers support students to learn about those practices. Below is the first page of the **Mathematical Practices in This Unit** essay for Grade 4 Unit 4, which highlights **MP5.**

CURRICULUM UNIT 4, PP. 8–11

MATHEMATICAL PRACTICES IN THIS UNIT

The eight Mathematical Practices are a critical part of students' mathematics learning. Mathematical Practice Notes are included throughout the unit to indicate opportunities for engaging students in these practices. Each unit focuses specifically on two Mathematical Practices.

In this unit, the highlighted practices are MP5, Use appropriate tools strategically, and MP6, Attend to precision. This essay describes each of these practices and provides examples from the unit of how to engage Grade 4 students in them.

MP5 Use appropriate tools strategically.

Mathematically proficient students at the elementary grades consider the tools that are available when solving a mathematical problem, whether in a real-world or mathematical context. These tools might include physical objects (cubes, geometric shapes, place value manipulatives, fraction bars, etc.), drawings or diagrams (number lines, tally marks, tape diagrams, arrays), paper and pencil, rulers and other measuring tools, scissors, tracing paper, grid paper, virtual manipulatives or other available technologies. Proficient students are sufficiently familiar with tools appropriate for their grade and areas of content to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained from their use as well as their limitations.

Mathematically proficient students choose tools that are relevant and useful to the problem at hand. These include such tools as are mentioned above as well as mathematical tools such as estimation or a particular strategy or algorithm.

(Illustrative Mathematics, *Standards for Mathematical Practice: Commentary and Elaborations for K–5*)

There are at least three different kinds of tools that are important in this unit: 1) tools for measuring length, area, and angle (ruler, yardstick, meterstick, grid paper, color tiles, protractor); 2) the set of Power Polygons, used for classifying polygons and for measuring and comparing angles; and 3) benchmarks and estimation, used in conjunction with measuring tools to anticipate measurements and ensure that they are reasonable.

In Session 1.3, students have started working on the activity, Perimeters Around the Classroom (*Student Activity Book*, page 173). Ursula and Steve are about to measure the perimeter of the doorway to the hall. The teacher asks them to estimate first:

Ursula: I'm 4 feet 2 inches, and I think the doorway is about twice my height, so that's around 8 feet for the two sides, and the part that goes across is about half of that, so that would be 4 and 4. I think the perimeter is 8 + 8 + 4 + 4, that's three 8s, about 24 feet.

Steve: That sounds about right because my dad is 6 feet, and he would come almost up to the top, well maybe a foot from the top. Actually, I'd say a little less than Ursula, maybe 7 and 7 for the sides and 3 and 3 for the top and bottom, so that's 20 feet.

Object	Unit of Measure (inches, feet, yards, centimeters, or meters)	Estimate	Actual Measurement
Your classroom door		20 feet	
Your teacher's desk			
The board			

UNIT 4: Measuring and Classifying Shapes | 8 | Mathematical Practices in This Unit

Mathematical Practices	Highlighted MPs in Unit…							
	1	2	3	4	5	6	7	8
MP1 Make sense of problems and persevere in solving them. In *Investigations,* this is a core practice essential to all of students' work. Throughout their work, students are coming to understand that mathematics makes sense and developing the confidence and skills to think through unfamiliar and difficult problems.	●						●	
MP2 Reason abstractly and quantitatively. This practice also focuses on making sense of problems, in particular, moving back and forth between abstract ideas and ways to embody those ideas with images and contexts. Students use story contexts, pictures, and physical models to ground their thinking and also learn to abstract quantities from those contexts and representations in order to reason about them numerically.			●				●	
MP3 Construct viable arguments and critique the reasoning of others. As students learn how to explain their solutions and justifications so that they communicate clearly to others, they are clarifying their own mathematical understanding. Analyzing and engaging with their classmates' explanations also deepens students' learning.		●				●		
MP4 Model with mathematics. When students write an equation to represent a story context, they are modeling with mathematics. When they create a table or graph to capture aspects of a set of data, they are modeling with mathematics. Students learn to use mathematics to capture the relationships among mathematical elements of real-world or fantasy situations.		●						●
MP5 Use appropriate tools strategically. Throughout their work, students learn to use a repertoire of tools as they solve problems. These include, for example, physical models, pictures and diagrams, measurement tools, and sets of geometric shapes. Students should have access to a range of tools and learn to select which can help them investigate a particular problem.				●	●			
MP6 Attend to precision. Students work on precision in computation, in measurement, and in articulating conjectures, explanations, and arguments. Constructing clear explanations, both written and oral, provides the opportunity for learning and using mathematical notation and vocabulary.				●		●		
MP7 Look for and make use of structure. In looking for structure, students are working with foundational ideas that underlie the mathematics they are learning, such as the structure of the base-10 system or the classification of polygons. Seeing structure involves recognizing the same mathematics in different contexts and equivalences among different mathematical objects.			●		●			
MP8 Look for and express regularity in repeated reasoning. Much of the practice of mathematics is not about solving one individual problem after another, but about looking across sets of related problems. In *Investigations,* students learn to pay attention to commonalities that lead to general ideas about mathematical objects and relationships.	●							●