The Foundations of Place Value

Understanding the place value structure of our base-10 number system, and how it applies to and supports number composition and computation, is a central piece of work in the number and operations strand of the *Investigations* curriculum. The base-10 number system is a place-value system. That is, any numeral, say 2, can represent different values, depending on where it appears in a written number: it can represent 2 ones, 2 tens, 2 hundreds, 2 thousands, and so forth. Understanding this system requires coordinating the way we write numbers—e.g., 217—and the way we name numbers in words—two hundred seventeen—with how the value of each place represents quantities.

In order to successfully work with place value, students need to know that one ten is equal to ten ones, and to be able to coordinate groups of tens and ones. Consider, for example, the number 32. One aspect of understanding the value of each place in this number is knowing that the 3 represents 3 groups of ten, the 2 represents 2 ones, and this can be expressed as $30 + 2$. It is also important for students to understand 32 as 2 groups of ten and 12 ones ($20 + 12$). The compactness of the base-10 system is what makes it powerful, but that very compactness means it is dense with ideas which young students must put together. That learning begins in Kindergarten.

The major focus of the work in Kindergarten is on counting and developing fluency with ones. This includes practicing the rote counting sequence, counting and counting out sets of objects, thinking about whether the quantity remains the same when a set is reorganized or counted in a different way, and looking at the way numbers are written.

As students are developing fluency with counting by ones, they are also developing an understanding about how numbers are composed. Six objects can be separated into four objects and two objects; that is, 6 can be composed as 4 and 2. Eight, the number that follows 7, is composed of 7 and 1.

Kindergarteners come to understand 10 as an important number, particularly through their use of Ten Frames. While initially introduced as an organizational tool for counting, the structure of the Ten Frame reinforces the foundational idea that ten ones can also be thought of as a *group* of ten ones. In their work with Ten Frames, students explore two-addend combinations of 10.

Toward the end of Kindergarten, students work with teen numbers, numbers from 10 to 19. They organize these quantities with Ten Frames and cubes to see them as ten ones and some leftover number of ones (which might be 0). When they record this information with equations, such as $15 = 10 + 5$ and $16 = 10 + 6$, they begin to notice important regularities in the structure of these numbers and the written numeral: the 1 in 15 refers to the group of ten ones and the 5 refers to the number of leftover ones. As students recognize the same ten-and-some-ones structure of the teen numbers in different contexts and representations (e.g., using Ten Frames to explore two-addend combinations of 10, and using more than one Ten Frame to explore the composition of numbers over 10), their understanding of that structure deepens.

The two aspects of place value emphasized in Kindergarten—combinations that make 10 and the composition of teen numbers—are structures students will use when they begin to add numbers whose total is greater than 10. For example, to add $8 + 6$, students might say, "I need 2 to make 10 so I'll take it from 6, leaving 4; $8 + 2 = 10$ and $10 + 4 = 14$." These are the first steps toward understanding place-value decomposition of larger numbers and using such decompositions in whole-number calculation strategies for all four operations—requirements for the computational fluency developed in later grades.