Extending Place Value to Tenths and Hundredths

Students have spent a great deal of time in elementary school working with the place value of whole numbers. In this unit, students focus on extending their understanding of the place-value system to tenths and hundredths.

The base-10 number system (also called the *decimal number system*) is a *place-value* system; that is, any numeral, such as 2, can represent different values, depending on where it appears in a written number: it can represent 2 ones, 2 tens, 2 hundreds, 2 thousands, and so on. Understanding a place-value system requires coordinating the way we write the numerals that represent a particular number (e.g., 5,217) and the way we name numbers in words (e.g., five thousand, two hundred seventeen) with how those symbols represent quantities.

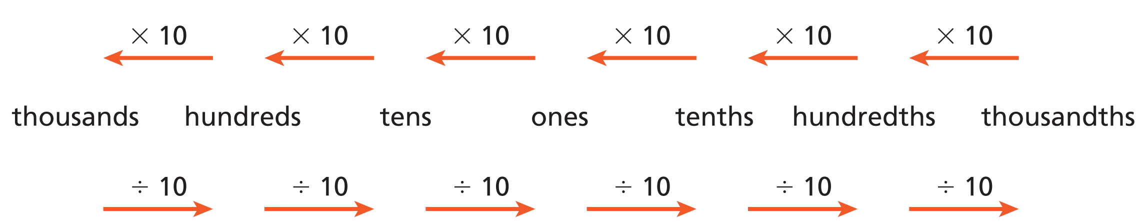
Numbers that include amounts less than one are represented as a continuation of the base-10 system with numerals to the right of the decimal point. The decimal point separates the integer and fractional parts of the number. The structure of the place-value system continues to hold true for these digits to the right of the decimal point. A digit in any place represents a value ten times greater than the same digit in the place immediately to the right and one-tenth of the value of the same digit in the place immediately to the left:

222.22
two hundred twenty-two and twenty-two hundredths

From left to right, the digits represent 2 hundreds, 2 tens, 2 ones, 2 tenths, and 2 hundredths. The digit in the hundreds place represents a quantity ten times greater than the digit in the tens place. The digit in the tenths place represents a quantity ten times greater than the digit in the hundredths place.

One of the problems for students in understanding decimals is that the amounts represented by tenths, and even more so by hundredths, are very small. They are much more experienced with whole numbers in their lives. Just as very large numbers are difficult to visualize, so are very small numbers. Students do not generally deal with hundredths in their everyday experience or, when they do, it is difficult to actually perceive the magnitude of those numbers. For example, 0.1 second and 0.01 second both represent a very short amount of time—the difference between them is not perceptible without specialized tools. It is perhaps easier to distinguish between 0.1 mile and 0.01 mile, but these are not quantities that come up in students' everyday experience. Smaller amounts—0.001 second or mile—are even more difficult to imagine. Yet understanding that 0.1 is ten times bigger than 0.01 is crucial to understanding and computing with decimals.

This same relationship holds true for whole numbers, such as 50 and 5 (50 is 10 times bigger than 5, 5 is one tenth of 50), but both the change in magnitude and the actual size of the numbers are more obvious in real situations: there is a noticeable difference between 50 seconds and 5 seconds. Even though the structure of the place-value system is the same for digits that indicate the fractional part of the number, experience in classrooms is that students need time to develop an understanding of the quantity, order, and equivalence with regard to these very small numbers.



The decimal point is the conventional separator used in the United States to separate the integer part of a number from parts of the number that are less than 1. One of the first understandings students should be developing is that digits to the right of the decimal point indicate that the quantity is greater than the whole-number portion of the number and less than the next largest whole number. For example, consider a swimmer who swims the 100-meter freestyle in 51.34 seconds. The part of the number represented by "0.34" indicates a fractional amount: it took more than 51 seconds for the swimmer to complete the race, but less than 52 seconds.

Learning about numbers that include digits to the right of the decimal point is further complicated by the way we read the numbers. The decimal portion of the number above is conventionally read as "twenty-two hundredths" rather than "two tenths and two hundredths." Students may at first read numbers in this second way, which actually indicates their understanding of the place value of each digit. Gradually students learn that two tenths plus two hundredths is equivalent to twenty-two hundredths and that the number is read in this way. This number is also conventionally read as "two hundred twenty-two point twenty-two," and students should recognize this as a correct way to read decimals. However, it is important that students say "twenty-two hundredths" to help them understand the value of decimals and make the connection between decimals and fractions.

The use of zeroes in decimal notation can be confusing to students. Students should sometimes see decimal numbers that are less than 1 written with a zero in the ones place, for example, 0.5. Including the zero helps to remind students that the decimal point separates the whole number portion of the number (which, in this case, is zero) from the part that is less than 1. However, students should also see numbers such as .5 written without the zero in the ones place, so that they recognize that $0.5 = .5$.

Students also learn about zeroes in places to the right of nonzero digits, such as $0.5 = 0.50$. Students are learning that five tenths (0.5) and fifty hundredths (0.50) are equal amounts. (Note that mathematically 0.5 and 0.50 are the same number; however, in statistics, science, and engineering, writing 0.5 or 0.50 may suggest a different level of confidence in the accuracy of results.)

In this unit, squares divided into tenths and hundredths are used as representations to help students visualize the relationship of these small numbers to one another and to 1. Students have been using representations similar to these to understand large numbers (for example, the 10,000 chart they use in Unit 5). Now, instead of building up a hundred, then a thousand, then 10,000, from single square units, they start with a single square unit and break it into smaller and smaller parts. What is critical is that students think of this square unit as one whole and the divisions of the square as equal parts of that one whole.