Addition Strategies

Students' strategies for addition fall into two basic categories: (1) Breaking the numbers apart and then adding these parts, and (2) Changing the numbers to numbers that are easier to add. In order to use these strategies, students must understand the meaning of addition and have a good mental model of what is happening in the problem. They need to look at the problem as a whole, think about the relationships between the numbers in the problem, and choose an approach they can carry out easily and accurately.

In the first category of strategies, students decompose numbers to take advantage of the base-10 structure of the number system. They either decompose numbers by place or decompose numbers to create or add a multiple of 10 or 100. In either case, when they add the parts together, they implicitly use either the commutative or associative property of addition, or both. That is, multiple addends can be combined in any grouping and in any order, as long as each addend is included exactly once.

At the end of this unit, Grade 4 students should be familiar with strategies in each category. They should feel comfortable and confident with more than one strategy and should be able to use it fluently—for example, by adding the largest or most logical parts of the number and using the fewest number of steps. Students should also be able to fluently use the U.S. standard algorithm for addition.

Below are examples of students' strategies for solving the following problem:

$$349 + 175 =$$

Although the steps for each strategy are all written out in this Teacher Note, in practice students gradually learn to carry out many of these steps mentally, jotting down just what they need to keep track of partial sums.

Breaking the Numbers Apart

In strategies that involve breaking numbers apart and then adding the parts, students use their understanding of the ways in which numbers can be decomposed to solve the problem.

In the solutions in Set A, students break both numbers apart by place value, add like places, and then find the final total. This approach is called "adding by place."

Set A: Adding by place

$$349 + 175 =$$

Solution 1

$300 + 100 = 400$

$40 + 70 = 110$

$9 + 5 = 14$

$400 + 110 + 14 = 524$

Solution 2

$$\begin{array}{r} 349 \\ +\ 175 \\ \hline 400 \\ 110 \\ 14 \\ \hline 524 \end{array}$$

Solution 3

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 3\,4\,9 \\ +\ 1\,7\,5 \\ \hline 5\,2\,4 \end{array}$$

The first two students added by place, starting with the largest place. When adding $40 + 70$, students may quickly add 30 and 70 first and then add 10. Students should be fluent with combinations of two numbers that add to 100 and use these mentally to add numbers with a sum greater than 100. Note that the only real difference in these two solutions is notation; in both solutions the students added by place, starting from the left. The third student used the U.S. standard algorithm for addition, which is also an example of adding by place, and starts from the right. Students compare Solutions 2 and 3 in Session 1.4.

In Set B, students break up one of the addends into parts and then add these parts, one at a time, to the other number. This approach is called "adding one number in parts."

Set B: Adding one number in parts

$$349 + 175 =$$

Solution 1	Solution 2
$349 + 100 = 449$	$349 + 100 = 449$
$449 + 70 = 519$	$449 + 50 = 499$
$519 + 5 = 524$	$499 + 25 = 524$

Solution 3

$349 + 51 = 400$

$400 + 100 = 500$

$500 + 24 = 524$

These three students started with the 349 but broke up the 175 in different ways (i.e., $100 + 70 + 5$, $100 + 50 + 25$, $51 + 100 + 24$). The first two students focused on how to break 175 into familiar chunks. The third student selected the first part to reach a landmark and then figured out what still needed to be added on.

When adding one number in parts, students should be encouraged to add the largest "chunks" possible while still making sense of the problem.

Changing the Numbers

In this category of approaches to solving addition problems, students change one or both of the numbers to what they often call "landmark" or "friendly" numbers, generally multiples of 10 or 100. In Set A, students change the numbers to multiples of 10 to create easier addition problems.

Set A: Changing the numbers and adjusting

$$349 + 175 =$$

Solution 1	Solution 2
$350 + 175 = 525$	$349 + 200 = 549$
$525 - 1 = 524$	$549 - 25 = 524$

Solution 3

$350 + 200 = 550$

$550 - 25 - 1 = 524$

After students change one or both numbers and find the sum, they have to decide what to do to the sum to compensate for their initial changes. The first student added 1 to 349 to make it easy to add to 175 and then had to subtract that 1 to get the final answer. The second student used a similar strategy, adding 25 to 175 and then subtracting 25 at the end. The third student changed both numbers, added them, and then subtracted the 25 and the 1 that had been added at the beginning.

Set B: Creating an equivalent problem

$$349 + 175 =$$

Solution 1	Solution 2
$324 + 200 = 524$	$400 + 124 = 524$

Sometimes students change the numbers in an addition problem in such a way that they create an equivalent problem that is easier to solve. In these examples, an increase in one number is matched by an equal decrease in the other number, so no additional adjustment is needed after the total has been found. The first student subtracted 25 from 349 and then added 25 to 175; the second student added and then subtracted 51. Students study this strategy in Session 1.3, not only because it is sometimes useful for solving addition problems, but also because it provides an opportunity for them to consider the properties of addition and to engage in the important processes of articulating, representing, and justifying generalizations.

Using Addition Strategies with 4- and 5-Digit Numbers

As students add numbers with 4 and 5 digits in Investigation 3, they realize the strategies described above continue to work, no matter the size of the numbers. Students are encouraged to look at the numbers being added and choose one of the strategies, including the U.S. standard algorithm for addition, which they can fluently use to solve any addition problem.