Subtraction Strategies

Students' strategies for subtraction fall into four basic categories: (1) Subtracting in parts, (2) Adding up or subtracting back from one number to the other, (3) Changing the numbers to numbers that are easier to subtract, and (4) Subtracting by place. To use these strategies, students must understand the meaning of subtraction and have a good mental model of what is happening in the problem. They must be able to look at the problem as a whole, think about the relationships between the numbers in the problem, and choose an approach they can carry out easily and accurately.

At the end of this unit, students in Grade 4 should be familiar with strategies in each category. They should feel comfortable and confident with at least one strategy and should be able to use it fluently—working with the largest or most logical parts of the number and using the fewest number of steps. Students should also be able to fluently use the U.S. standard algorithm for subtraction.

Here are examples of students' strategies for solving the following problem:

$$451 - 287 =$$

Subtracting in Parts

Solution 1	Solution 2
$451 - 200 = 251$	$451 - 200 = 251$
$251 - 80 = 171$	$251 - 50 = 201$
$171 - 7 = 164$	$201 - 30 = 171$
	$171 - 7 = 164$

Solution 3

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  451
- 200
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  251
-  50
-----
  201
-  30
-----
  171
-   7
-----
  164
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These three students subtracted 287 in parts. The first student broke up 287 by place (200 + 80 + 7), and the other two students broke the 87 into different parts (50 + 30 + 7). Note that the only real difference between Solutions 2 and 3 is the notation. As students use this strategy, encourage them to subtract the largest parts they can while still making sense of the problem and the numbers. Work with students to gradually subtract larger amounts; for example, subtracting 80 rather than subtracting 50 and then 30. However, keep in mind that fluent students often quickly subtract smaller parts mentally without the need to write down all the steps. Students sometimes call this strategy "subtracting one number in parts."

Adding Up and Subtracting Back

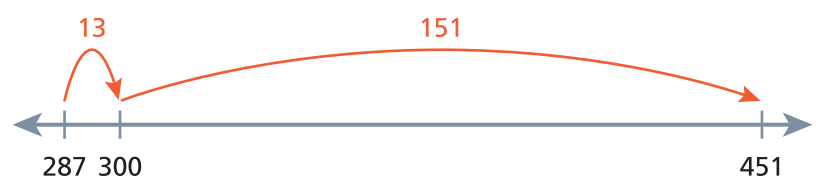
In this category of strategies, students visualize how much more or less one number is than the other and either "add up" or "subtract back" to find their answer. They often represent the subtraction as the distance between two numbers on a number line.

Set A: Adding up

In Set A, students start at 287 and "add up" until they reach 451.

$$451 - 287 =$$

Solution 1	Solution 2
$287 + \underline{13} = 300$	$287 + \underline{100} = 387$
$300 + \underline{151} = 451$	$387 + \underline{60} = 447$
$13 + 151 = 164$	$447 + \underline{4} = 451$
	$100 + 60 + 4 = 164$

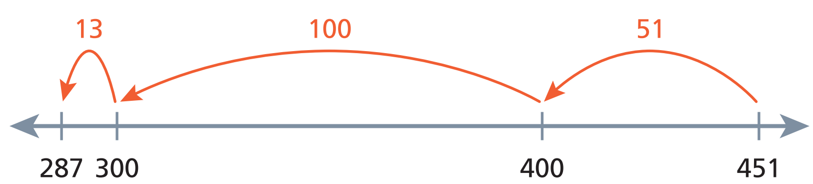


Both students thought of the solution as how much must be added to 287 to get a sum of 451. Implicitly, they are using the inverse relationship of addition and subtraction to solve the problem. As shown on the number line, the first student added 13 to 287 to get to 300 and then added 151 to get to 451. The second student looked for the largest multiple of 100 and then the largest multiple of 10 that could be added. Students often call this strategy "adding up."

Set B: Subtracting back

In this set of solutions, students start at 451 and then subtract back until they reach 287.

Solution 1	Solution 2
$451 - \underline{51} = 400$ $400 - \underline{100} = 300$ $300 - \underline{13} = 287$ $51 + 100 + 13 = 164$	$\begin{array}{r} 451 \\ -\ 151 \\ \hline 300 \\ -\ 13 \\ \hline 287 \end{array}$ $151 + 13 = 164$



Both students solved the problem by "going back" to 300 and then "back 13 more" to 287. As you can see in the number line representation, students are not subtracting 287 in parts as in the first category; rather, they start at 451, subtract until they reach 287, and then determine how much they subtracted. Students often describe this method as figuring out "how far" 287 is from 451. (Note that this strategy is typically used with word problems and a number line, and students use it less frequently as numbers start getting larger.)

Changing the Numbers

In this group of strategies, students change one or both of the numbers to what they often call "landmark" or "friendly" numbers.

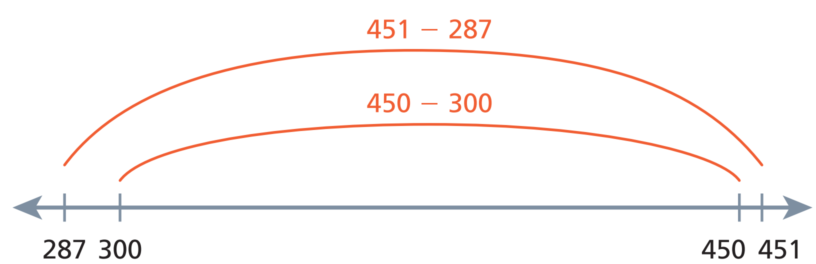
Set A: Changing a number and adjusting

In Set A, students change one or both of the numbers, subtract, and then compensate for the changes they made.

$$451 - 287 =$$

Solution 1	Solution 2
$451 - 300 = 151$	$450 - 300 = 150$
$151 + 13 = 164$	$150 + 14 = 164$

The first student changed 287 to 300 to create an easier subtraction problem. Because 13 too many had been subtracted, 13 was added to 151 to get the final answer. The second student changed both numbers and had to decide how both of those changes affected the result. The difference between the two numbers was decreased by 1 (changing the 451 to 450) and by 13 (changing the 287 to 300), so 14 was added to 150. In both solutions, the changes made the difference smaller. Visualizing the effect of the changes and how to compensate for those changes is critical to using this kind of strategy successfully. Number lines are particularly useful tools for showing how changing numbers affects the result, as the number line below illustrates for Solution 2.



In general, students should not be encouraged to change both numbers in a subtraction problem, as in Solution 2, because this is not a useful strategy for most students. If a student can indeed visualize what these changes mean and how to adjust the result to get the answer to the original problem, then that student can certainly use this method when it makes sense for a particular problem. However, too many students change both numbers to "easy" numbers without a clear idea of how those changes affect the difference. Classroom experience indicates that thinking through how to adjust the result after changing only one number in a subtraction problem can be challenging for Grade 4 students. Discussions of this idea (as in Session 2.6) are fruitful in helping students think more deeply about the operation of subtraction and the relationship between the two numbers in a subtraction expression, whether or not students actually continue to use this method to solve problems.

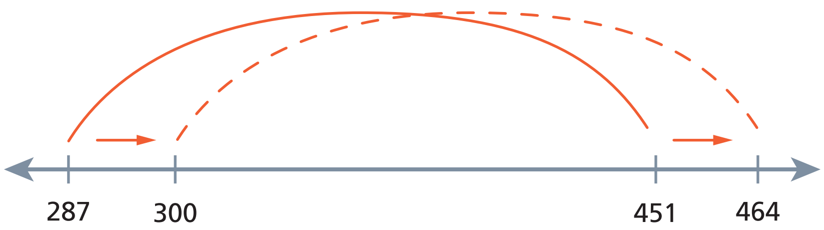
Set B: Creating an equivalent problem

The next student's strategy is an example of changing both numbers in order to create an equivalent problem that can then be solved without any need to compensate for changes.

Solution 1

$$451 - 287 = 464 - 300 = 164$$

This student created an equivalent problem that is easier to solve by adding 13 to both numbers. Adding (or subtracting) the same quantity to (or from) both numbers maintains the difference. You may visualize transforming the problem in this way as "sliding" the difference along a number line.



Although it is not surprising for students to use this strategy, it is not as common as the others listed here. It is not explicitly mentioned in the text of Investigation 2, but it may come up in your classroom, especially for problems for which changing the numbers by small amounts creates a much easier problem—for example, $454 - 399 = 455 - 400 = 55$.

Subtracting by Place

Adding by place—adding 1s to 1s, 10s to 10s, 100s to 100s, and so on—is one of the addition strategies most often used by students. However, subtracting by place is not as straightforward. Consider the problem $451 - 287$. It is easy to subtract 200 from 400, but how do you subtract 80 from 50 or 7 from 1? The following strategies are based on subtracting by place value. The solutions below are the "borrowing," or regrouping, algorithm—which has been, and continues to be, commonly taught in the United States. This algorithm requires recomposing the number 451 to make it possible to subtract in each place. The shorthand notation shown below means that 451 ($400 + 50 + 1$) has been recomposed into $300 + 140 + 11$, which then allows easy subtraction by place. (This algorithm is studied in Investigation 2.)

Solution 1

$$\begin{array}{r} {}^{14} \\ {}^{3}\;\not{}^{4}\;{}^{11} \\ \not4\;\not5\;\not1 \\ -\,2\;8\;7 \\ \hline 1\;6\;4 \end{array}$$

Solution 2

$$\begin{array}{r} {}^{1} \\ {}^{3}\;{}^{4}\;{}^{1} \\ \not4\;\not5\;1 \\ -\,2\;8\;7 \\ \hline 1\;6\;4 \end{array}$$

Note that the only difference in these two solutions is the notation. Both students had to regroup the tens and hundreds places. Solution 1 shows the notation used in this unit, where the digit is crossed out to show that its value in that place has changed (so each digit 4, 5, and 1 is crossed out). In Solution 2, the student only crossed out the digit to show regrouping has happened (the 4 and 5), and then wrote a small "1" next to the 1 and next to the 4. Either solution is acceptable as the U.S. standard algorithm for subtraction.

This algorithm ensures that, in each place, the top number is always equal to or greater than the bottom number so that the difference in that place is positive. Some students understand that it is possible in mathematics to subtract a larger number from a smaller number, getting a negative result. You may see these students use an algorithm in which they subtract by place without regrouping, getting both positive and negative results that they then combine:

$$\begin{array}{r} 451 \\ \underline{-\,287} \\ -\,6 \\ -\,30 \\ \underline{200} \end{array}$$

$$200 + (-30) + (-6) = 164$$

Since most Grade 4 students do not have experience with negative numbers, time is not spent on this algorithm in this unit; but you should be aware of how it works in case a student uses it.

Using Subtraction Strategies with 4- and 5-Digit Numbers

As students subtract numbers with 4 and 5 digits in Investigation 3, they realize that the strategies described above continue to work, no matter the size of the numbers. Students are encouraged to look at the numbers being subtracted, and choose one of the strategies they can fluently use to solve any subtraction problem.