Images of Multiplication

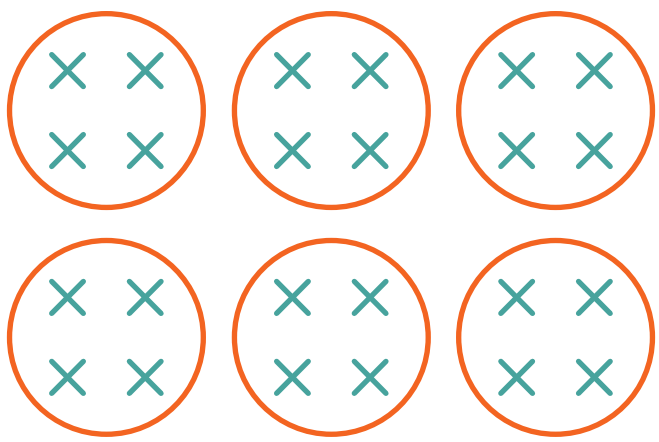
It is important that students develop strong visual images of multiplication as they develop strategies for solving multiplication problems. If students can visualize clearly how the numbers they are multiplying are related, they are able to keep in mind the meaning and behaviors of multiplication as they develop flexible, efficient, and accurate strategies for solving multiplication problems.

Students encounter many ways to represent multiplication in Grade 3—pictures of groups of things in a story context, skip counting on a 100 chart, rectangular arrays, and as the area of a rectangle. As students work with larger numbers in Grades 3 and 4, it will become cumbersome to draw pictures, skip count on a large number chart, or use arrays with all the individual units shown. Based on their experience creating these representations with small numbers, students learn to visualize these representations mentally and use them to break up the numbers appropriately and keep track of which parts of the problem have been solved and which remain to be solved.

As you work with students, suggest these ways of representing multiplication. Over time, as students become more practiced in creating and using these images on paper, ask them to begin to use mental images of, for example, a story context or an array to help them figure out where to start or what part of a problem has been solved and how to continue.

Images of Equal Groups in a Story Context

In this unit, students learn to represent a multiplication expression such as 6×4 by creating a picture similar to this one:



Ask students to generate simple stories that help them visualize a multiplication expression such as 6×4 as equal groups; for example, six bags with four marbles in each bag. Help students select contexts that are familiar to them. Then you can ask students to imagine that context as a way of thinking through the problem. In Grade 3, students are moving away from thinking of multiplication as repeated addition. Instead of adding up 4s, students can be encouraged to use the image to start with a larger chunk of the problem. For example, you might ask, "Can you visualize how many marbles would be in two bags? In three bags? Now how many more bags of four are there?" A story context involving equal groups can help students use what they know to determine the product: "I know that there are 12 marbles in three of the bags, and there are three more bags, so I double that to get 24."

Representing Multiplication as Skip Counting

In this unit, students mark off multiples of the numbers 2–6 and 10 on 100 charts.

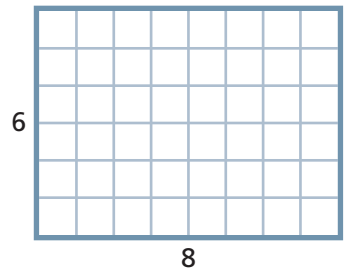
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

These charts provide an opportunity for students to notice patterns in each number's multiples and to consider the relationship between multiples of various numbers (for example, that multiples of 4 are also multiples of 2). Ask students questions that help them visualize the counting

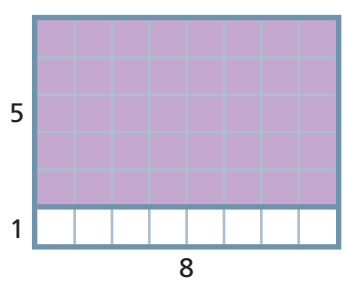
number sequence and think through how to calculate the next multiple as they are skip counting. For example, "You said that you've counted by 8s to 48. What is a way of quickly figuring out what 8 more will be?" (For example, add 2 to get to 50, then 6 more; or add 10 to get to 58, then subtract 2.) If students use skip counting to solve a less familiar multiplication fact, encourage them to start with a known multiple and then continue to skip count rather than skip counting from the beginning each time. For example, to solve 8×6, students may know that $8 \times 5 = 40$, and then make one more jump of 8 to 48.

Representing Multiplication with Arrays

In this unit and Unit 5, students work with Array Cards and drawings in which all the individual units of the array are visible. The rows of each rectangular array represent equal groups and the number of squares in each row represents the amount in each group. Arrays lay an important foundation for students in understanding how dimensions of a rectangle can be multiplied to find the area. Students study the idea of area further in Unit 4.



This 6×8 array can be seen as 6 groups of 8 items or, if we consider the columns to be the groups, 8 groups of 6 items. In either case, students can visualize the problem as a whole and the smaller parts that may help them to find the product; for example, if juice boxes come in sets of 8, a student might think of 6×8 as 6 sets of juice boxes. The student could visualize these in an array and use that image to break the problem into parts that are easier to solve.

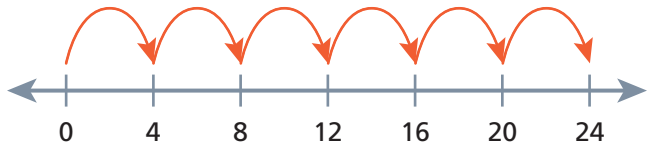


$5 \times 8 = 40$, $1 \times 8 = 8$, $40 + 8 = 48$

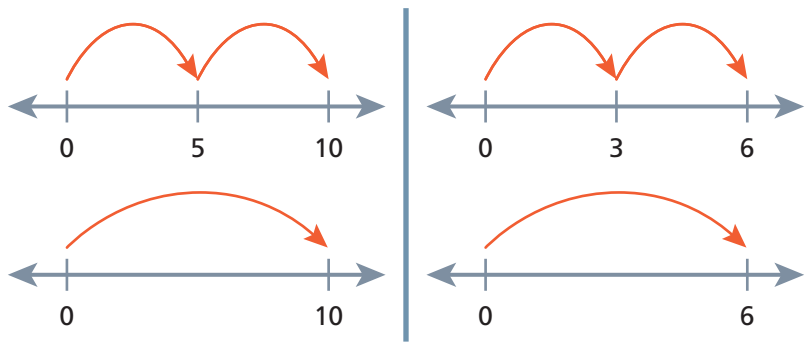
Visualizing how to break multiplication problems into parts becomes even more important as students solve multidigit problems later in Grade 3, and in Grades 4 and 5. See **Teacher Note 6:** Representing Multiplication with Arrays for more information about how arrays are used in Grade 3 and how the use of arrays can be extended to represent more difficult multiplication and division problems.

Representing Multiplication with the Number Line

In earlier grades, students used the number line to show addition and subtraction. Some students who are comfortable using the number line for addition may also find it helpful to use it for multiplication. Skip counting on the number line clearly shows the accumulation of equal groups.



This representation may be particularly useful when students are working with doubling and halving in Investigation 2. For example, as students consider the relationship between multiples of 5 and 10 or multiples of 3 and 6, some students may use the number line to justify their arguments.



See Kathryn's reasoning in **Dialogue Box 1:** Relationships Between the Multiples of 5 and 10.