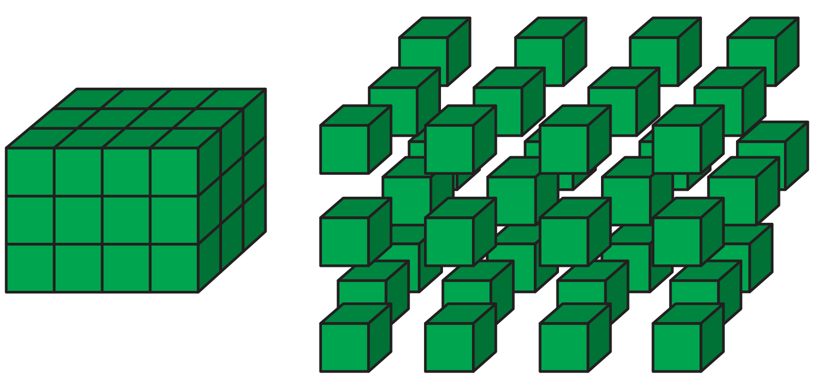
Strategies for Finding the Number of Cubes in 3-D Arrays

For students to determine how many cubes are in a 3-D array, they must mentally construct an image or model of the set of cubes. Students have been observed doing this in a variety of ways, some of which are listed below in increasing order of the students' ability to see the whole and its parts in an organized manner. You may see your students progress from less organized to more organized methods as the unit advances. Some students may approach different tasks with different levels of understanding. As students gain more experience with ideas about volume, they begin to pay more attention to the dimensions of 3-D solids and use these dimensions in volume formulas. The conceptual work that students do with cube arrays at the beginning of this unit is critical to developing an understanding of the structure of 3-D solids, so that volume formulas are used with understanding.

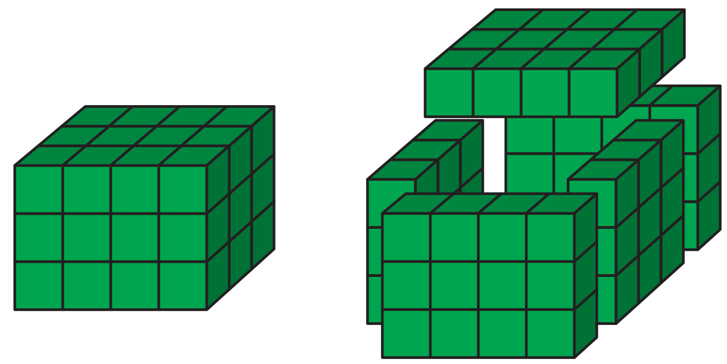
Seeing Arrays as Unstructured Sets

Whether given an actual cube package (e.g., a rectangular prism), a picture of a cube package, or the box that contains that package, some students do not see any organization. In this case, students usually count cubes one by one and almost always lose track of their count. For these students, the task is like counting a large number of randomly arranged objects.



Seeing Arrays in Terms of Sides or Faces

Many students approach 3-D arrays of cubes by thinking only about the sides of the rectangular prism formed by the cubes. These students might count all or some of the cube faces that appear on the six sides. With this method, edge cubes are often counted more than once and cubes in the middle are missed.



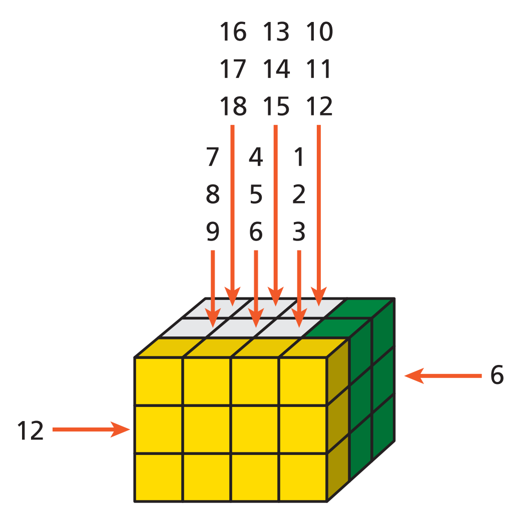
Using this approach, a box of 36 cubes, 3 × 4 × 3, might be counted as 54 cubes—the front, back, and top each as 12, the right and left sides each as 9.

Most students with this "sides" conceptualization use it consistently, whether they are looking at pictures of boxes, box patterns, or the actual cube configurations. Students who see cube arrays in terms of their faces do not necessarily think of arrays as hollow; they simply think that their method counts all the cubes inside and out.

Seeing Arrays as Having Outside and Inside Parts

Students who take this approach try to count both the outside and the inside of the 3-D array, sometimes doing it correctly but more often incorrectly. They attempt to visualize the entire package and account for each cube.

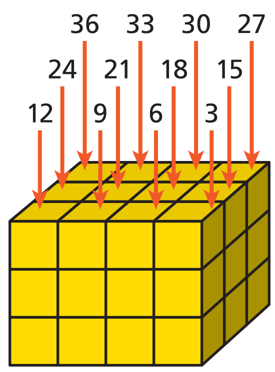
Correct Counting One student counted the cubes visible on the front face (12) and then counted those on the right side that had not already been counted (6). She then pointed to the remaining cubes on the top, and for each, counted cubes in columns of 3: 1, 2, 3; 4, 5, 6; . . . ; 16, 17, 18. She then added 12, 6, and 18 for a total of 36.



Incorrect Counting One student counted all the outside cube faces of this same array, getting 66. He then said that there were 2 cubes in the middle for a total of 68.

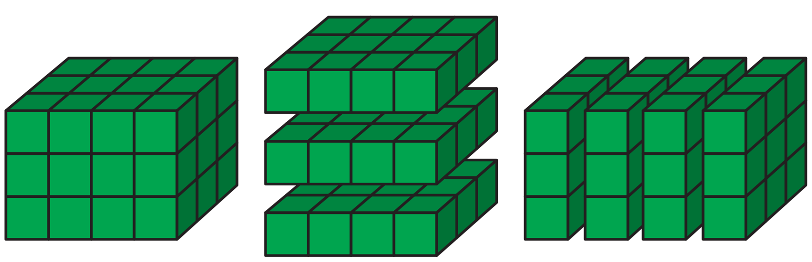
Seeing Arrays in Terms of Rows or Columns

Students count the cubes in successive rows or columns by ones or by skip counting. In the strategy diagrammed below, the student counted 3 cubes for each of the 12 columns.



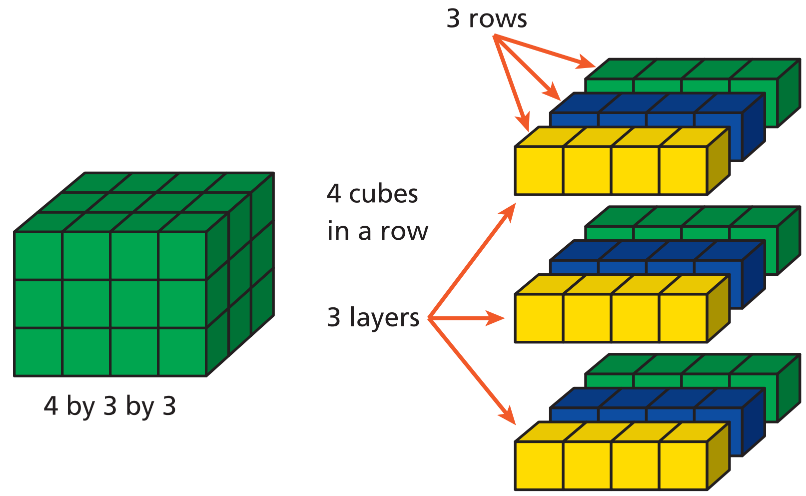
Seeing Arrays in Terms of Layers

Students determine the number of cubes in one layer and then multiply or use repeated addition to account for all the layers. The layers can be horizontal or vertical, and students often use one of the visible faces in a picture as a representation of a layer. Other students look at a box pattern, see the bottom as representing a layer, and then determine the number of layers by looking at the sides of the pattern. Many students who use layering often count the cubes in a layer one by one.



Seeing Arrays as Layers Described by Dimensions

Some students understand how dimensions can be used to describe and count the cubes in an array.



Students might reason that the length gives the number of cubes in a row and the width gives the number of rows in a layer, so the number of cubes in a layer is the product of the length and width. Because the height gives the number of layers, they multiply the number of cubes in a layer by the height to find the total number of cubes in the array. Initially, not all students refer to the dimensions as length, width, and height. They begin to use these descriptions more consistently when they are introduced to volume formulas in Session 1.5.

The Learning Process

Students gradually progress to more powerful ways of conceptualizing cube configurations and relating these cube arrays to ideas about the volume of rectangular prisms. They have repeated experiences with building boxes, filling them with cubes, determining the number of cubes in a box, and discussing their ideas with classmates. As the unit progresses, this work lays a foundation for understanding volume as a cubic measurement and to using the formulas $V = l \times w \times h$ or $V = b \times h$.