TEACHER NOTE 2

Visualizing Fraction Equivalencies

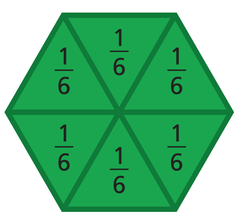
In this unit, students use materials and representations to build their knowledge of fraction equivalencies with halves, fourths, eighths, thirds, and sixths. Just as students build a repertoire of certain whole-number equivalencies (e.g., the single-digit addition combinations or the different ways in which 138 might be decomposed), they also build a repertoire of fraction equivalencies.

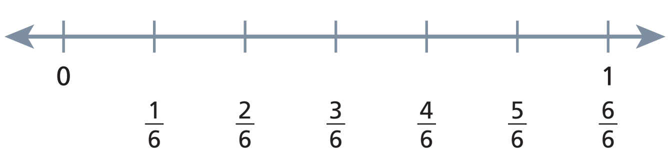
In this unit students work on fraction equivalents, including:

- Fractions that equal whole numbers $\left(\frac{2}{2}, \frac{3}{3}, \frac{6}{6}, \frac{4}{4}, \frac{8}{8} \text{ and } \frac{2}{1}, \frac{3}{1}, \text{etc.}\right)$
- Fractions that equal $\frac{1}{2}$ $\left(\frac{2}{4}, \frac{3}{6}, \frac{4}{8}\right)$
- Other equivalent fractions $\left(\frac{1}{3} = \frac{2}{6} \text{ and } \frac{1}{4} = \frac{2}{8}\right)$

They also work on addition combinations that are equivalent to 1 or to another fraction $\left(\text{e.g., } \frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1 \text{ and } \frac{1}{3} + \frac{1}{6} = \frac{1}{2}\right)$.

Throughout this unit, students use pattern blocks, paper rectangles, number lines, and drawings to help them develop visual images of how fractions or combinations of fractions are equivalent. They come to understand that two halves, three thirds, and six sixths make a whole because they see the whole is completely "filled" with fractional pieces or because it takes two steps of $\frac{1}{2}$, three steps of $\frac{1}{3}$, or six steps of $\frac{1}{6}$ to get from 0 to 1 on the number line.





Similarly, students learn equivalents for $\frac{1}{2}$. They visualize how these fractions relate to the whole or to 1 and may notice that these fractions (e.g., $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}$, and so on) are characterized by a denominator that is twice the numerator.

Equivalent fractions such as $\frac{1}{4}$ and $\frac{2}{8}$ become familiar to students as they first fold and cut, and eventually draw, rectangles to represent the whole and the equal parts that are the fractions of that whole. For example, students quickly realize that when fourths are divided in half, two of the new smaller pieces equal one fourth. In working with number lines, students also find that fractions such as $\frac{2}{8}$ and $\frac{1}{4}$ are equivalent, because they are located at the same spot on the number line.

Students develop a repertoire of these common equivalents by reasoning about the numbers and by visualizing these fractions and combinations of fractions represented as parts of a whole or on the number line. For example, students often come away from this unit knowing that $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$ because they have a picture in their minds of pattern block pieces that represent $\frac{1}{3}$ and $\frac{1}{6}$ and how those pieces relate to the piece that represents $\frac{1}{2}$. This work prepares them for work in later grades on adding fractions with unlike denominators.

In Grades 1–3, students have been working on putting together and taking apart whole numbers. As they develop flexibility in decomposing numbers, they learn about the way our number system is structured, and they develop computational methods based on this understanding. In Grades 3–5, students engage in developing this same kind of flexibility with fractions and decimal fractions. By the end of this unit, students are using the structure of known fraction equivalencies to reason about other equivalencies (e.g., "Three sixths plus one half must be a whole because three sixths is a half, and that makes two halves"). Understanding the meaning of fractions and developing a repertoire of equivalencies are two of the key components that lay the groundwork for operations with fractions.

Comparing Fractions

Listen to students comparing fractions as they work on these problems during Investigation 2. Students develop a number of strategies that work for certain kinds of comparisons. You can help students become more explicit about the underlying regularities they are noticing as they use these strategies by asking questions such as these:

"How did you know which fraction was greater? Will that strategy always work? What types of fractions does your strategy work for? Can you give me another example of a comparison of two fractions when this strategy would work?"

Explicit discussions about comparing fractions are built into Sessions 2.4 and 2.5. But you can ask students about these ideas whenever they come up.

"Can you come up with a rule about your strategy—how it works and what kinds of fractions it works for?"

You may want to start a list of conjectures, worded by the students, based on what students are noticing. Here are some examples of strategies they develop and conjectures they might articulate, given the general ideas underlying each strategy.

Comparing Fractions to 1

Example: $\frac{3}{4}$ and $\frac{4}{4}$

"I know that $\frac{4}{4}$ is equal to 1 because it's all of the fourths from 0 to 1. $\frac{3}{4}$ has to be less."

Conjecture: If one of the fractions has the same top and bottom numbers, then it's always 1. A fraction with a smaller number on top and a larger number on the bottom has to be smaller than 1.

Example: $\frac{3}{4}$ and $\frac{3}{2}$

"To get to three halves you have to go past 1—$\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$,—and three fourths isn't even one."

Conjecture: If a fraction has a numerator that is less than the denominator (such as $\frac{3}{4}$), it is less than one. If a fraction has a numerator that is more than the denominator (such as $\frac{4}{3}$), it is greater than one, so it's greater than the other fraction.

Comparing Fractions to $\frac{1}{2}$

Example: $\frac{2}{6}$ and $\frac{2}{4}$

"$\frac{2}{4}$ is equal to $\frac{1}{2}$. If the ant is going by sixths, it takes six steps to get to 1, so it takes 3 steps to get to $\frac{1}{2}$, so two sixths is smaller because it only has two of the $\frac{1}{6}$ steps."

Conjecture: If the numerator of a fraction is less than half the denominator, the fraction is less than $\frac{1}{2}$.

Comparing Fractions with the Same Denominator

Example: $\frac{5}{8}$ and $\frac{2}{8}$

"All the moves on the number line are the same size. So if you have five, of course it's more than two."

Conjecture: When the denominators are the same, the fraction with the larger numerator is larger.

Comparing Fractions with the Same Numerator

Example: $\frac{3}{8}$ and $\frac{3}{4}$

"The fourths are bigger than the eighths. So if you go three of the fourths, you go further than three of the eighths, so the three fourths has to be bigger."

Conjecture: When the numerators are the same, the fraction with the smaller denominator is larger.