

The eight Mathematical Practices are a critical part of students' mathematics learning. Mathematical Practice Notes are included throughout the unit to indicate opportunities for engaging students in these practices. Each unit focuses specifically on two Mathematical Practices.

In this unit, the highlighted practices are MP1, Make sense of problems and persevere in solving them, and MP8, Look for and express regularity in repeated reasoning. This essay describes each of these practices and provides examples from the unit of how to engage Grade 5 students in them.

MP1 Make sense of problems and persevere in solving them.

Mathematically proficient students at the elementary grades explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. For example, young students might use concrete objects or pictures to show the actions of a problem, such as counting out and joining two sets to solve an addition problem. If students are not at first making sense of a problem or seeing a way to begin, they ask questions that will help them get started. As they work, they continually ask themselves, "Does this make sense?" When they find that their solution pathway does not make sense, they look for another pathway that does. They may consider simpler forms of the original problem; for example, to solve a problem involving multidigit numbers, they might first consider similar problems that involve multiples of ten or one hundred. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. They often check their answers to problems using a different method or approach.

Mathematically proficient students consider different representations of the problem and different solution pathways, both their own and those of other students, in order to identify and analyze correspondences among approaches. They can explain correspondences among physical models, pictures or diagrams, equations, verbal descriptions, tables, and graphs.

(Illustrative Mathematics, Standards for Mathematical Practice: Commentary and Elaborations for K–5)

In this unit, students work with representations and contexts to understand multiplication that involves fractions and decimals. While students come into this unit knowing a great deal about multiplication, their experience has been with whole numbers. They are used to thinking about multiplication as involving some number of equal whole-number groups. And, they are used to multiplication problems in which the product is greater than either of the factors. In order to understand multiplication with fractions, students need to apply their prior knowledge of multiplication while also thinking through why multiplication involving fractions does not, at first, appear to have all the features of multiplication that they have come to expect.

Perseverance in solving problems requires working with prior conceptions, while also expanding those conceptions to accommodate new mathematics. In order to do this, teachers need to give students opportunities to make explicit how they are thinking about a new type of problem. In Session 1.4, students have been solving problems that involve multiplication of a fraction or mixed number by a whole number. After sharing solutions and representations, the teacher asks them to think through how and why these problems are multiplication.

Teacher: When we started working on these problems, some of you weren't sure these were really multiplication problems. I'm curious as to what you are thinking now. What makes these multiplication problems?

Shandra: Multiplication is like equal groups, and so these are like $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$. It's pretty much the same thing.

Janet: At first, I wasn't thinking of $\frac{1}{2}$ as a group. But then I was thinking about the race problems. You can say 7 groups of 2, like if you ran 2 miles every day. But if you run $\frac{1}{2}$ mile a day, you can still do 7 times $\frac{1}{2}$. So I agree with Shandra.

Walter: Also, you can split up what you're multiplying just like you do with whole numbers.



- Teacher:** What do you mean that you can split them up?
- Walter:** In the problem about $2\frac{1}{4}$ miles for 5 days, you can do 5 times 2 and then 5 times $\frac{1}{4}$. Just like if it was 24 and you might split it into 20 and 4.
- Teacher:** So you've talked about a couple of ways that multiplying with fractions doesn't seem different than multiplying with whole numbers. But I know some of you had some thoughts about how some of these problems didn't seem like multiplication. Zachary, could you talk about what you and Lourdes were saying about Problem 2? Would one of you read the problem and then tell us what you were thinking?
- Zachary:** [reading the problem] Martin is going to ride in the Great Bicycle Race, which is $\frac{7}{10}$ as long as the 100-mile Astounding Race. How long is the Great Bicycle Race?
- Lourdes:** Yeah, so I was saying that it's not like groups. It's not like what Shandra was saying with $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ or something like that. It's just $\frac{7}{10}$ and then 100. It's more like division. You're dividing the 100 into tenths and then taking 7 of them.
- Stuart:** We were thinking about that, too, and Yumiko said to me that it's not a whole group. It's like 100 is the group and you're taking $\frac{7}{10}$ of that.
- Mitch:** I think that's why it's multiplication. You can do 2 groups or 3 groups or a part of a group.
- Teacher:** We'll be working on division with fractions later in this unit, so we'll come back to this discussion then. Before we stop for today, does anyone else have a thought about why these problems you've been working on are multiplication?

The teacher knows that not all students are settled in their thinking about why these problems are multiplication, and she will provide more opportunities to continue this discussion. Discussions like this one help students make sense of these new problems. Pausing to consider what students' confusions have been and how they are thinking about them as the unit proceeds gives students the time to listen to other students' explanations as they continue to think through their own understanding. Hearing that others are working through difficult ideas, just as they are, encourages students to persevere in their attempts to understand, represent, and articulate new ideas.

The following chart shows where Mathematical Practice Notes specifically address MP1 and when that mathematical practice is assessed.

MP1 Make sense of problems and persevere in solving them.		
SESSION	MPN	ASSESSMENT CHECKLIST
1.1	●	
1.4	●	
1.7	●	
1.11	●	
2.1	●	
2.3	●	
3.2	●	
3.4		●
3.5	●	●
3.6	●	
3.8	●	
3.10	●	