Strategies for Comparing Fractions

Listen to students comparing fractions as they solve the problems on *Student Activity Book* pages 142–143 and pages 147–148. There are many strategies that students use for comparing fractions. In Grade 4, students kept track of these strategies and listed conjectures about strategies for comparing certain types of fraction pairs on a class chart. As you observe students work, discuss their strategies with them, and encourage them to articulate their reasoning and the underlying regularities they are noticing. Some students may remember some of these strategies from their work in Grade 4. Ask questions such as these:

- **For what types of fractions does your strategy work?**
- **Can you give me another example of a comparison of two fractions when this strategy would work?**
- **Can you come up with a rule about your strategy—how it works and for what kinds of fractions it works?**

Here are some examples, expressed in students' words, of strategies they develop and conjectures they may articulate, given the general ideas underlying each strategy.

Comparing Fractions with the Same Denominator

Example: $\frac{3}{12}$ and $\frac{2}{12}$

Yumiko: All the pieces are the same size. So, if you have 3, of course it's more than 2.

Conjecture: When the denominators are the same, the fraction with the larger numerator is greater.

Comparing Fractions with the Same Numerator

Example: $\frac{3}{8}$ and $\frac{3}{4}$

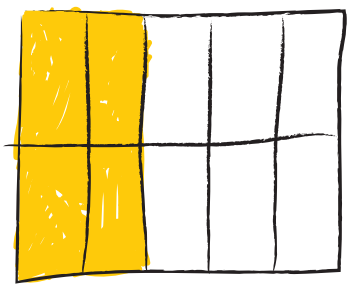
Renaldo: Three eighths is less than $\frac{3}{4}$ because 4 parts is less than 8, so the fourths are bigger than the eighths. So if you have the same number of parts, but the fourths are bigger, then $\frac{3}{4}$ has to be bigger.

Conjecture: When the numerators are the same, the fraction with the smaller denominator is greater.

Comparing Fractions in Which the Numerator and Denominator of One Fraction Are Double Those of the Other

Example: $\frac{4}{10}$ and $\frac{2}{5}$

Tavon: In my rectangle, you can see that this shaded part is $\frac{4}{10}$, but if you just make them into fifths, you have to use two of the tenths for a fifth, so you cut the number of pieces in half, and you cut the denominator in half because now there are only 5 pieces, not 10.



[Tavon's Work]

Conjecture: If the numerator and denominator of one fraction $\left(\text{such as } \frac{4}{10}\right)$ are double the numerator and the denominator of another fraction $\left(\text{such as } \frac{2}{5}\right)$, the fractions are equivalent.

Comparing Fractions to 1

Example 1: $\frac{4}{5}$ and $\frac{4}{3}$

Joshua: There is one whole in $\frac{4}{3}$, with some still left over, and $\frac{4}{5}$ isn't even 1.

Conjecture: If a fraction has a numerator that's less than the denominator $\left(\text{such as } \frac{4}{5}\right)$, it's less than 1. If a fraction has a numerator that's more than the denominator $\left(\text{such as } \frac{4}{3}\right)$, it's greater than 1

Example 2: $\frac{4}{5}$ and $\frac{7}{8}$

Samantha: $\frac{7}{8}$ is only $\frac{1}{8}$ away from 1. But $\frac{4}{5}$ is $\frac{1}{5}$ away from 1. An eighth is smaller than a fifth, and so $\frac{7}{8}$ is just a little smidge from 1. $\frac{7}{8}$ is closer to 1, so it's bigger.

Conjecture: If two fractions are just one piece less than 1, the fraction with smaller pieces is greater.

Comparing Fractions to $\frac{1}{2}$

Example 1: $\frac{2}{5}$ and $\frac{1}{2}$

Benito: If there are 5 pieces in the rectangle, $\frac{1}{2}$ of the rectangle has $2\frac{1}{2}$ pieces. One half has two and a half fifths, so $\frac{2}{5}$ is smaller because it has only two fifths.

Conjecture: If the numerator of a fraction is less than half the denominator, the fraction is less than $\frac{1}{2}$.

Example 2: $\frac{2}{5}$ and $\frac{3}{8}$

Shandra: For $\frac{3}{8}$, you need another $\frac{1}{8}$ to make a half. For $\frac{2}{5}$, you need half of a fifth to make a half. That's the same as $\frac{1}{10}$. $\frac{1}{10}$ is smaller than $\frac{1}{8}$, so $\frac{2}{5}$ is closer to $\frac{1}{2}$. That means that $\frac{2}{5}$ is more.

Conjecture: If you have two fractions that are less than $\frac{1}{2}$ and you look at the missing piece to get to $\frac{1}{2}$, the fraction with the smaller missing piece is greater than the other fraction.

Finding equivalent fractions and comparing fractions to landmarks are important building blocks as Grade 5 students learn to add and subtract fractions with unlike denominators.