What's the "Difference"?

Teachers must be aware of the various ways students may interpret ideas during mathematical discussions. The fact that some words have multiple meanings can be tricky for students, particularly so for English Language Learners. These multiple meanings exist within everyday English usage, but such meanings are even more difficult for students to learn when there are differences between how a word is used in the everyday vernacular and how that same word is used in a mathematical sense. For example, in everyday usage, the word similar means "resembling, without being identical" as in "John and Jose have similar haircuts." In mathematics, on the other hand, the word similar is a geometric term that refers to shapes "having the same shape, with the same angles and proportions, though not necessarily of the same size."

As students develop mathematical terminology, they continue to refine their understanding of the mathematical meaning of particular words. As they do so, teachers need to understand the multiple registers in which their students move to facilitate communication, understand their work, and more accurately assess their understanding of mathematical concepts.1

In the following case, fifth-grade teacher Gretchen Hopkins shares with us an English Language Learner's confusion around the word difference.

Carmelo is a quiet and shy fifth-grade student who has been in this school since kindergarten. He speaks Spanish at home. Many new things are happening for him this year: He no longer receives ELL services in school, and he is working with the Investigations curriculum for the first time.

I have noticed that when I try to engage Carmelo in mathematical discourse, he is not able to use much math terminology and usually ducks his head in embarrassment. When I do get him to explain his ideas, he mostly talks around the subject, and I end up guessing his meaning and trying to fit it into the discussion that is going on. When I ask Carmelo questions to confirm my understanding of what he means, he invariably nods in agreement. I'm getting the

sense more and more that he will agree with anything I say as long as he can then hop off the hot seat. Just like the teacher in Judit Moschkovich's article, I think that there is sometimes a discrepancy between what he is trying to articulate and what I am assuming he is referring to.

A clear example of that discrepancy occurred when Carmelo was working on a homework assignment involving the Digits Game. The students were given three different sets of digits; they selected numbers from each set to write a number as close as possible to 2,500. Carmelo was able to arrange the digits to make numbers that were reasonably close (though not necessarily the closest) to 2,500:

He used 1, 2, 6, 1, and 3 to make 2,611.

He used 2, 9, 0, 6, and 4 to make 2,604.

He used 8, 9, 4, 3, and 3 to make 3,438.

However, when he was asked to determine the difference between his numbers and the target, 2,500, he wrote:

6, 11

6.4

3, 4, 3, 8

I was confused by this response until I realized that Carmelo was not interpreting the word difference in the mathematical sense of subtraction but was thinking about it in terms of the common, everyday sense of "not alike." He wrote all of the digits that were different from, that is, "not like," the digits in 2,500.

Even after 5 years of English instruction in school, there are many discrepancies between what I think Carmelo understands and what he actually understands. Is he confused about the vocabulary, the concept, or both? In examining Carmelo's work through this new lens, I realize that I need to make accommodations to address the confusions that arise for him as he negotiates between the mathematical and everyday registers in Spanish and English and between the

Moschkovich, Judit. 1999. Supporting the participation of English language learners in mathematical discussions. In For the Learning of Mathematics, vol. 19, no. 1, 11-19.

two languages. For example, in the activity described here, my plan is to do the following:

- See if Carmelo can solve these problems if I rephrase the questions to clarify the term *difference* and say, for example, "How much more is 2,604 than 2,500?"
- Explain to Carmelo how I intended to use the word difference in this context.
- Ask him to place the number he wrote and the target number of 2,500 on a number line and connect the two to give him an image of the difference between the two numbers.

I need to help Carmelo move to a clearer understanding of mathematical vocabulary as it relates to his understanding of math concepts. I am going to have to make a more concerted effort to put him at ease in our math discussions so he will take more risks in describing his ideas and in the process develop more fluency with math terminology.

As Mrs. Hopkins took the time to understand Carmelo's unexpected response to a homework activity, she realized that he didn't understand the intention of the question he was asked. This insight into Carmelo's mistake allowed Mrs. Hopkins to become more aware of the multiple ways students understand and use math terminology. While English speakers go through the process of negotiating between the everyday and mathematical registers, among English Language Learners the process is more complex. Not only do English Language Learners need to negotiate between these two registers in English; they also need to do so between the two registers in their first language and between their first language and English.