Teacher Note

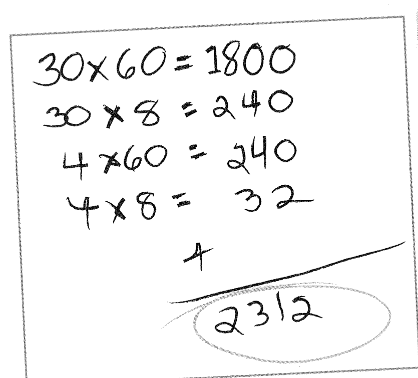
Computational Fluency and Place Value

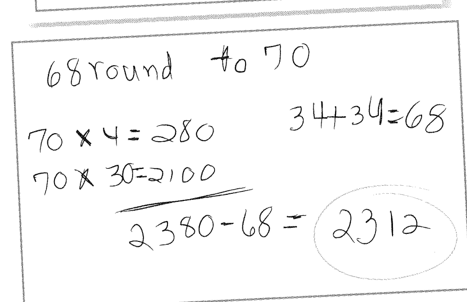
Computational fluency includes accuracy, flexibility, and efficiency. When fluency with a particular operation is achieved, students can look at the problem as a whole, choose a solution strategy that they can carry out easily without becoming bogged down or losing track of their steps, use their strategy to solve the problem accurately, recognize whether the result is reasonable, and double-check their work. Students who are fluent have a repertoire that includes mental strategies, strategies in which only intermediate steps are jotted down while other steps are carried out mentally, and strategies that require a complete written solution. They are flexible in their choice of algorithm or procedure, and they can use one method to check another.

Developing computational fluency with whole numbers is central to the elementary curriculum. This development includes the building blocks of computation:

- Understanding the base-ten number system and its place value notation
- Understanding the meaning of the operations and their relationships
- Knowing the basic addition and multiplication number combinations (the "facts") and their counterparts for subtraction and division
- Estimating reasonable results
- Interpreting problems embedded in contexts and applying the operations correctly to these problems
- Learning, practicing, and consolidating accurate and efficient strategies for computing
- Developing curiosity about numbers and operations, their characteristics, and how they work
- Learning to articulate, represent, and justify generalizations

At each grade level, computational fluency looks different. Students are progressing in learning the meaning of the four arithmetic operations with whole numbers, developing methods grounded in this meaning, and gradually solving problems of greater difficulty through the grades. At each grade level, benchmarks for whole number computation indicate what is expected of all students by the end of each curriculum unit and each grade, although work at each grade level goes beyond these benchmarks. Gradually, approaches to problems become more efficient, flexible, and accurate. For example, in Grade 1, many students begin the year adding by direct modeling of the problem with objects and counting the sum by ones. By the end of the year, students are expected to start with one of the quantities and count on the other, and for some combinations students "just know" the sum or use known combinations to solve others ("I know $4 + 4 = 8$, so $4 + 5 = 9$"). In Grade 4, many students start the year solving some multiplication problems by skip counting, but by the end of the year, they are expected to solve multidigit multiplication problems such as 34×68 by breaking problems into subproblems, based on the distributive property.





Sample Student Work

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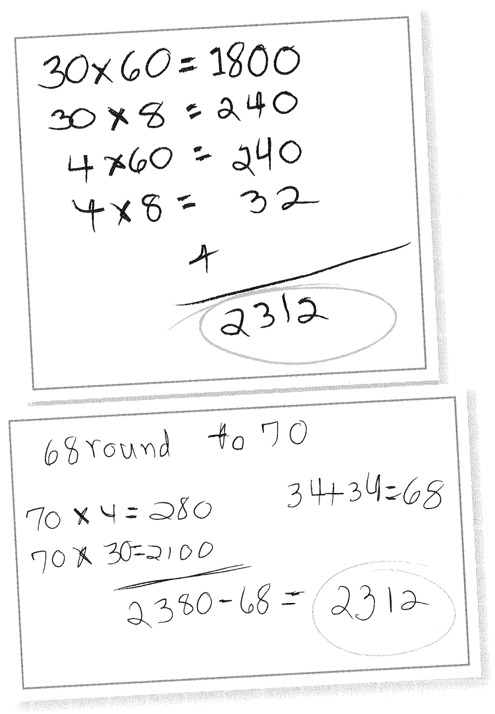

Sample Student Work

Understanding the Base-Ten Number System

Learning about whole number computation is closely connected to learning about the base-ten number system. The base-ten number system is a "place value" system. That is, any numeral, say 2, can represent different values, depending on where it appears in a written number: it can represent 2 ones, 2 tens, 2 hundreds, 2 thousands, as well as 2 tenths, 2 hundredths, and so forth. Understanding this place value system requires coordinating the way we write the numerals that represent a particular number (e.g., 217) and the way we name numbers in words (e.g., two hundred seventeen) with how those symbols represent quantities.

The heart of this work is relating written numerals to the quantity and to how the quantity is composed. It builds from work on tens and ones in Grades 1 and 2 to a focus on numbers in the hundreds and thousands in Grade 3, and work with numbers in the ten thousands, hundred thousands, and beyond in Grades 4 and 5. Knowing place value is not simply a matter of saying that 217 "has 2 hundreds, 1 ten, and 7 ones," which students can easily learn to do by following a pattern without attaching much meaning to what they are saying. Students must learn to visualize how 217 is built up from hundreds, tens, and ones, in a way that helps them relate its value to other quantities. Understanding the place value of a number such as 217 entails knowing, for example, that 217 is closer to 200 than to 300, that it is 100 more than 117, that it is 17 more than 200, that it is 3 less than 220, and that it is composed of 21 tens and 7 ones.

A thorough understanding of the base-ten number system is one of the critical building blocks for developing computational fluency. Understanding place value is at the heart of estimating and computing. For example, consider adding two different quantities to 32:

$32 + 30 =$ _____

$32 + 3 =$ _____

How much will 32 increase in each case? Students think about how the first sum will now have 6 tens, but the ones will not change, whereas in the second sum, the ones will change, but the tens remain the same. Adding three *tens* almost doubles 32, while adding three *ones* increases its value by a small amount. Considering the place value of numbers that are being added, subtracted, multiplied, or divided provides the basis for developing a reasonable estimate of the result.

The composition of numbers from multiples of 1, 10, 100, 1,000, and so forth, is the basis for most of the strategies students adopt for whole number operations. Students' computational algorithms and procedures depend on knowing how to decompose numbers and knowing the effects of operating with multiples of 10. For example, one of the most common algorithms for addition is adding by place. Each number is decomposed into ones, tens, hundreds, and so forth; these parts are then combined. For example,

$326 + 493$

$300 + 400 = 700$

$20 + 90 = 110$

$6 + 3 = 9$

$700 + 110 + 9 = 819$

To carry out this algorithm fluently, students must know a great deal about place value, not just how to decompose numbers. They must also be able to apply their knowledge of single-digit sums such as $3 + 4$ and $2 + 9$ to sums such as $300 + 400$ and $20 + 90$. In other words, they know how to interpret the place value of numbers *as they operate with them*—in this case, that just as 2 ones plus 9 ones equals 11 ones, 2 tens plus 9 tens equals 11 tens, or 110.

As with addition, algorithms for multidigit multiplication also depend on knowing how the place value of numbers is interpreted as numbers are multiplied. Again, students must understand how they can apply knowledge of single-digit combinations such as 3×4 to solve problems such as 36×42.

For example,

36×42

$30 \times 40 = 1{,}200$

$30 \times 2 = 60$

$6 \times 40 = 240$

$6 \times 2 = 12$

$1{,}200 + 240 + 60 + 12 = 1{,}512$

Students gradually learn how a knowledge of 3×4 helps them solve 30×4, 3×40, 30×40, 3×400, and so forth.

Building Computational Fluency Over Time

There is a tremendous amount of work to do in the area of numbers and operations in Grades K–5.

- Kindergartners and first graders are still working on coordinating written and spoken numbers with their quantitative meaning.
- Second graders are uncovering the relationship between 10 ones and 1 ten and between 10 tens and 1 hundred.
- Third graders are immersed in how the properties of multiplication differ from the properties of addition.
- Fourth and fifth graders are solving multidigit problems and becoming flexible in their use of a number of algorithms.

This list provides only a brief glimpse of how much work there is to do in these grades.

Students gain computational fluency in each operation through several years of careful development. Extended time across several grades is spent on each operation. Students build computational fluency with small numbers as they learn about the meaning and properties of the operation. Then they gradually expand their work to more difficult problems as they develop, analyze, compare, and practice general methods.

Let's use subtraction as an example of this process:

- In Kindergarten and Grade 1, students solve subtraction problems by modeling the action of subtraction.
- By Grade 2, students are articulating and using the inverse relationship between addition and subtraction to solve problems like the following: "If I have 10 cookies, how many more cookies do I need to bake so I have 24?"
- During Grades 2 and 3, students become fluent with the subtraction "facts" and model and solve a variety of types of subtraction problems, including comparison and missing part problems. By Grade 3, as students' understanding of the base-ten number system grows, they use their understanding of place value to solve problems with larger numbers.
- In Grades 3 and 4, students articulate, represent, and justify important generalizations about subtraction. For example, if you add the same amount to (or subtract it from) each number in a subtraction expression, the difference does not change, as in the equation $483 - 197 = 486 - 200$. In these grades, students also choose one or two procedures, practice them, and expand their command of these procedures with multidigit numbers.
- In Grades 4 and 5, as their fluency with subtraction increases, students analyze and compare strategies for solving subtraction problems. Because they are fluent with more "transparent" algorithms for subtraction in which the place value of the numbers is clear, they are now in a position to appreciate the shortcut notation of the U.S. traditional regrouping algorithm for subtraction, analyze how it works, and compare it to other algorithms. (See the Teacher Note, Computational Algorithms and Methods.)