Teacher Note

Why Are Fractions Difficult? Developing Meaning for Fractions

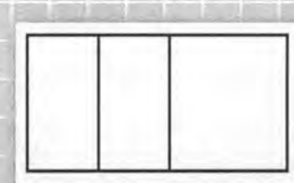
Why are fractions difficult for third graders? As adults, we are used to seeing and ascribing meaning to fractions such as $\frac{1}{2}$ and $\frac{3}{4}$ in a variety of situations. Imagine how strange this notation—two whole numbers separated by a line—must look to elementary school students as they begin to dig into the meaning of fractions.

It is not surprising, then, that in trying to understand fractions, students attempt to draw on what they know from their experience with the numbers with which they are most familiar—whole numbers. For example, students may think at first that $\frac{1}{3}$ is greater than $\frac{1}{2}$ because 3 is greater than 2. There has been a great deal of research conducted regarding students' understanding of fractions, largely with middle school students. This research indicates that even in the middle grades, many students still interpret fraction notation as two separate whole numbers that are not related.

Looking at fractions as though they represent two separate whole numbers leads to misinterpretation of their meaning and an inability to assess the reasonableness of calculation results. For example, an often-cited assessment question from the National Assessment of Educational Progress (NAEP) asked students to estimate the sum of $\frac{12}{13}$ and $\frac{7}{8}$. Given four answer choices—1, 2, 19, and 21—students most often chose 19 or 21. Using whole-number addition, they added either only the numerators or only the denominators. These students were not able to think of each of these numbers as being close to 1 and thus missed the correct estimate of 2.

In order to develop meaning for fractions, students work with the context of "fair shares," in which something is shared equally. One key idea in this unit—and throughout students' study of fractions—is that a fraction represents a quantity in relation to a unit whole. Examples of unit wholes could include a single object, an area, a linear measure, or a group of objects. In this context, "$\frac{1}{2}$" means "one out of two equal parts that make up one whole."

One half of one whole is not the same quantity as one half of another whole; for example, $\frac{1}{2}$ a class of 26 is 13 students, and $\frac{1}{2}$ a class of 22 is 11 students. However, although $\frac{1}{2}$ can represent many different quantities, depending on the size of the whole, $\frac{1}{2}$ has the *same relationship* to *any* whole. It is one of the two equal parts that compose the whole. In this unit, students work with fractions in relation to a whole that is a single object ($\frac{1}{4}$ of one brownie), an area ($\frac{2}{3}$ of the surface of a hexagonal pattern block), or a group of things ($\frac{1}{4}$ of six brownies). The focus is twofold: the parts of the whole must be equal to one another, and all the parts combined must equal the whole. It is not unusual for third graders to ignore one or both of these ideas at first. For example, when dividing brownies, they may make unequal pieces (as in the first picture) or cut off part of the whole in order to make the pieces equal (as in the second picture below).



The ideas in this unit also lay the groundwork for division of a smaller number by a larger one. Students sometimes think or are told by adults, "You can't subtract a larger number from a smaller one," when of course it is quite possible to solve such a problem when you know about negative numbers ($4 - 7 = -3$). Subtracting 7 from 4 requires expanding one's knowledge of the number system to include negative numbers. Similarly, they may think, "You can't divide a smaller number by a larger one."

However, dividing 3 by 4 requires expanding one's knowledge of the number system to include rational numbers (numbers that can be represented as a division of two integers, such as $\frac{1}{2}$ or $\frac{9}{10}$). In this unit, as students work on activities such as dividing seven brownies among four people, they implicitly apply the distributive property by thinking of $7 \div 4$ as $(4 \div 4) + (3 \div 4)$. They solve the first part of the problem by assigning one brownie to each of the four people and then tackle the second part of the problem, $3 \div 4$, by solving it in a number of ways (see **Dialogue Box:** Seven Brownies, Four People, page 120).

In fact, fractions indicate division: one interpretation of $\frac{1}{2}$ is that it represents one out of two equal parts of a whole, but it also means the quantity that results from dividing one by two. In Grade 3, students are not yet thinking about fractions as an indicated division. Instead, they are learning about a fraction as a *relationship* between two numbers and how that relationship is, in turn, related to 1. In Grade 4 they will focus more on how a fraction is a *number* that always has the same relationship to 1 and will extend the use of the number line with whole numbers to fractions, mixed numbers, and decimals.