Adding Two Evens or Two Odds

During Session 1.3, students have been thinking about what happens when you add different combinations of even and odd numbers. In this discussion at the end of the session, students use their definitions of odd and even as they justify what happens when *two odd* or *two even* numbers are combined. Notice that the teacher encourages students to explain their thinking by asking "Why?" and "Is that always true?"

Teacher: What did you find out about adding two even numbers?

Katrina: You can't make an odd by adding two evens.

Holly: You only get evens when you add an even plus an even.

Jeffrey: Yes; when we added two evens, we got an even.

Carla: It has to be even.

Teacher: Why do you say it has to be even, Carla?

Carla: Because an odd number has to have an extra number, and there isn't an extra number so it has to be even.

Darren: That's what we got, too. If the two numbers are even, it's always even.

Teacher: But does anyone have an idea why?

Katrina: Because it can't be odd. There's no way.

Jeffrey: If it's even, you can break it into 2s [takes cube towers of 6 and 4 and breaks them into pairs], 2, and 2, and 2, and 2, and 2. If you put them together or take them apart, it's still all 2s.

Teacher: What did you find out about adding two odds?

Gregory: If you plus an odd and an odd, you'll always get an even.

Anita: That's right. When we added 3 and 9, we got 12. That's even.

Darren: It's always even, no matter what you do. Everything we got was even.

Teacher: So Gregory, Anita, and Darren are saying that if you add two odds, you get an even. Is that always true? Can anyone explain why that might be true?

Jacy: With two odds—we did 7 and 5. It's really just like 6 and 6. If you take 1 off the 7 and put it on the 5 [demonstrates with cubes], it's all even again.

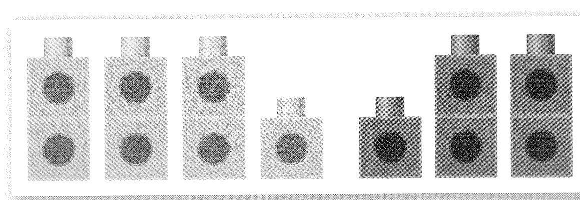
Teacher: So that's two odds making an even. Everyone build 7 and 5 with your cubes. Can anyone else add to what Jacy said?

Luis: It's like what Jeffrey said about 2s.

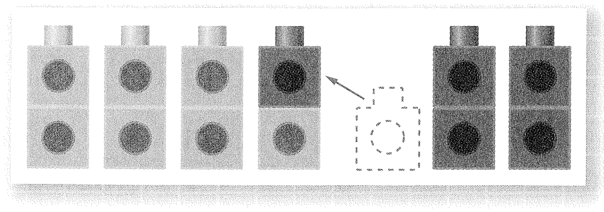
Tia: Even though an odd has a leftover, you can just squish them together.

Teacher: What do you mean by "squish them together"?

Tia demonstrates with cubes how you can break 7 into 3 groups of 2 with 1 leftover, and 5 into 2 groups of 2 with 1 leftover.



Tia: You can take the 2 leftovers and squish them together.



Teacher: Do you think that's true for all odds?

Tia: Yes!

Teacher: Why?

Tia: All odds have a leftover.

Simon: If you take two odd numbers, there are always 2 leftovers that can be partners.

Malcolm: I agree. It's like two evens. If you just had 6 and 4, it would be even, but there's 1 extra on each and that makes another 2, so it's still even altogether.

Notice that both parts of this conversation move from specific examples to general arguments aimed at explaining why the given generalization is true. The discussion about adding two evens moves from the unsuccessful search for two evens that add to an odd to arguments, such as Carla's and Jeffrey's, about why it is impossible for the sum of two even numbers to be odd. These arguments depend on the agreed-on definitions of even and odd numbers.

Similarly, the discussion about the sum of two odds moves from talk about specific instances to general arguments based on the one leftover from each odd number joining together to make another pair. Testing specific instances, as Anita did, is an important starting point. Such a test gives students a chance to state a generalization and offer evidence. The teacher's role is to encourage students to explain why it must always work that way.