

Teacher Note

Discussing Mathematical Ideas

Throughout the *Investigations* curriculum, whole-class discussion is a key aspect of students' mathematical activity. Class discussion provides a time for students to

- articulate their mathematical ideas.
- share different approaches to solving a problem.
- identify and investigate what they don't understand.
- analyze why a solution works or how it is flawed.
- pose conjectures and identify evidence to support them.
- collaborate to build ideas or solve problems.
- develop mathematical language.
- use representations to describe mathematical relationships.
- compare and connect students' various ideas, representations, and solutions.
- learn to consider and question each other's ideas.

By carefully selecting problems, representations, and solutions for the whole class to consider, the teacher focuses discussion on key mathematical ideas and works with the class as a whole to move students' thinking forward.

Building Mathematical Community

In the first weeks of school, teachers help the class develop norms for classroom discussion and work with students on attitudes and behavior that will support productive math discussions. Most teachers find that they need to work quite explicitly with students throughout the school year to first establish and then maintain expectations for class discussion. During discussions, teachers keep the flow of ideas organized and remind students about the appropriate focus. For example, "Right now I want comments that are either agreeing with, disagreeing with, or commenting on Yolanda's idea," or "So we now have three different approaches to this problem on the board. Is there a way in which Jill's is similar

to Corey's?" Teachers also find opportunities to comment directly on student actions, behavior, and contributions that support productive discourse:

Because Stephen was willing to talk through what was confusing him when he got an answer that he knew wasn't right, it seemed to really help all of us understand this kind of problem better.

When Kamala put up her picture of the problem, I heard some of you say, "Ooh!" What was it you understood when you saw that picture? Did anyone else have a picture or a diagram that helped you understand how to solve this problem?

And from time to time teachers discuss directly with the class what aspects of class discussions have been helping or hindering students' participation:

What helps you be willing to share your work or make an observation during class discussion? Are there times you don't feel comfortable speaking? Why is that?

Building an inclusive mathematics classroom involves a focus on respect for student ideas and acceptance of differences. Working on establishing this community with students will vary across grades and even from one year to another, depending on the needs and experiences of your students. (See the section "Setting Up the Mathematical Community" in Part 7, Working with the Range of Learners: Classroom Cases for some teachers' thoughts on building the classroom mathematics community.)

Focusing Class Discussions

Students' ideas are important and are, in fact, central to discussion. But if a discussion bounces among too many different ideas or tries to include too many different approaches, the discussion becomes ungrounded and hard for students to follow. Simply listing one problem-solving approach after another doesn't engage students beyond the few moments when they are contributing their own idea.

The Math Focus Points for Discussion and sample teacher dialogue found in the text for every discussion will help you guide the discussion. In preparing for class, ask yourself:

- What do I want this discussion to accomplish?
- What do I want all students to take away from this discussion?
- How will the time spent as a whole class enhance the work students have done individually or in pairs or groups?

During work that precedes the discussion, observe students' work with the upcoming discussion in mind. Ask yourself:

- What is a difficulty that many students are having?
- What is a problem that many students are struggling with?
- Is there a question that one pair or group came up with that it would be fruitful for the whole class to discuss?
- What are the basic approaches to solving this problem that students are using?
- Which students or groups have ideas or approaches that should be shared?

Student Participation

Whole-class discussion time is precious class time; it should serve to consolidate or move ahead the math thinking of all students. Find ways during discussions to elicit responses from different students. Although all students may not participate in any one discussion, all of your students' voices should be heard over the course of several discussions. There are many ways to work with students to encourage them to participate. For example, listen carefully to students' ideas and look carefully at their work during activities. Help particular students prepare to share one of their ideas. At first, some students might be more comfortable if you put their solution, representation, or idea on the board or a

transparency and present it to the class yourself; alternatively, the student might explain a certain part of the solution, while you add to the student's explanation.

Think of ways to invite all students' participation during each discussion by asking students to raise hands if they used the same approach or if they agree or disagree with a statement you or another student makes. Pose a question and have students discuss it for a few minutes in pairs before having the whole class consider it. Use wait time judiciously and think about ways that students can use quiet signals when they are ready to respond (e.g., thumbs up rather than hands waving); then students who are still thinking are not distracted.

Ideas are bound to come up that you cannot pursue during class discussions. Sometimes you cannot follow or decipher a student's idea at the moment or you are not sure about how it relates to what is being discussed. If you don't understand what a student is saying, you might ask another student to interpret or talk to the student later. Don't be afraid to let students know that you have to think about something and get back to them or follow up with them after the discussion. You can always bring an idea back to the class later if you decide it would be important for the class to think about it.

You can find other ways to follow up a student's idea that is not central or accessible for the whole class: "I was thinking about your idea, and here's a problem I thought you could try it on." Some teachers have a "parking lot" poster for ideas that come up during class but don't have time to pursue. These ideas may come up again later or can be referred to when they become relevant. The better you know the curriculum, the more you will know when they might come up.

Setting Up the Classroom for Discussion

It is critical that students are sitting in such a way that everyone is focused on the discussion and everyone can hear. If there are representations that students need to see during the discussions, they must be large enough and dark enough so that everyone can see them.

A variety of seating arrangements for class discussions can work, as long as there are clear expectations connected to them. In some classrooms, students gather on a rug for the class meeting and then return to their places or choose places for work time. In other classrooms, students often stay at their own desks for meetings. Some teachers vary the setting, with students staying at their desks when the meeting will be short and gathering together when a longer time is needed.

To facilitate a smooth transition to meeting on the rug, some teachers assign students places to sit on the rug, changing them every month or so. Others place circles, mats, or white boards to clearly mark the places available for students to sit. Others allow students to sit wherever they want in a circle as long as they can see the teacher and all of the other students. They might remind students to make a good choice about sitting in a position and next to classmates that enables them to focus on the discussion. While some students can pay attention while sitting on the floor, others do better in a chair.

Guidelines for Whole-Class Discussions

In summary, here are some guidelines to keep in mind for your class's whole-group discussions:

- Set up norms and review them frequently; point out examples in which they are working.
- Plan a clear purpose and focus for each discussion, based on the listed Focus Points.
- Use *wait time to give students time to think*.
- Ask students to use quiet student signals to indicate they are ready to respond.
- Prepare with some students ahead of time to participate in the discussion.
- Have clear visuals that everyone can see and refer to.
- Establish a routine arrangement that ensures that everyone can hear and see.
- Select only a few students to share solutions.

When all students come to a discussion prepared to listen actively and to contribute ideas, the class discussions provide an important forum in which they can articulate, represent, connect, and consolidate the mathematical ideas they have been working on.