Multiplication Strategies

Based on the first step students take, their strategies for multiplication fall into the following three basic categories:

1. Breaking the numbers apart by addition
2. Changing one factor and adjusting
3. Creating an equivalent problem

In the first strategy, students break the given numbers apart and create a series of partial products. In the second and third strategies, students change the problem in some way to create a problem that is easier to solve. Students often use a combination of these approaches when solving a single problem and may see their own variations or combinations as different strategies.

In order to use the strategies described in this **Teacher Note**, students need to understand the meaning of multiplication and have a good mental model of what is happening in the problem. They need to look at the problem as a whole, think about the relationships of the numbers in the problem, and choose an approach they can carry out easily and accurately.

Using the distributive property is essential in solving multiplication problems. When using this property, students can break the numbers in any multiplication problem into parts and then multiply each part of one number by each part of the other number(s). For example, if the problem is 48×42, they might think of 48 as $40 + 8$, and/or think of 42 as $40 + 2$. This breaking apart is usually done by place, but not always. For example, for the problem 27×18, the student might think of 27 as $25 + 2$. It is not necessary for students to use the term distributive property, or the notation $(40 + 8) \times (40 + 2)$. What is important is that they realize that the numbers can be broken apart by addition, that they know how to keep track of multiplying the parts, and that they add all the partial products to find the final product.

By the end of Grade 5, students should be fluent with one or more of these strategies, and the U.S. standard algorithm for multiplication. They should be able to work with the largest or most reasonable parts of the number, using a small number of steps in both solving problems and recording their solutions. They may use one strategy most of the time, but during the year they should encounter and understand all of these strategies.

Here are examples of the three strategies.

1. Breaking the numbers apart by addition

Many students choose to break the numbers apart by place and find all the partial products. Here are two ways a student might record this approach:

$$
\begin{array}{l}
48 \times 42 \\[1em]
\begin{array}{rr}
40 \times 40 = & 1{,}600 \\
40 \times \ \ 2 = & 80 \\
8 \times 40 = & 320 \\
8 \times \ \ 2 = & \underline{\quad 16} \\
 & 2{,}016
\end{array}
\end{array}
\qquad\qquad
\begin{array}{rl}
48 & \\
\underline{\times\ 42} & \\
1{,}600 & 40 \times 40 \\
320 & 40 \times \ \ 8 \\
80 & 2 \times 40 \\
\underline{16} & 2 \times \ \ 8 \\
2{,}016 &
\end{array}
$$

Note that because of the commutative and associative properties of multiplication and addition, numbers can be multiplied or added in any order.

For a simpler problem, students might break apart only one number. For example, to solve 22×13, a student might break apart only the 13, thinking of the problem as $(22 \times 10) + (22 \times 3)$, since both of these partial products are solved easily.

2. Changing one factor and adjusting

In this sample solution, a student changes 48×42 to 50×42, solves 50×42, and then compensates for the initial change.

$$
\begin{array}{l}
48 \times 42 \\
50 \times 42 = 2{,}100 \\
2{,}100 - 84 = 2{,}016
\end{array}
$$

Changing 48×42 to 50×42 results in a problem that some students find easier to solve. The new numbers can be either broken apart by addition, yielding $(50 \times 40) + (50 \times 2)$, or thought of as $\frac{1}{2}$ of 100×42. Then the student has to decide how to adjust the answer to 50×42. Because 50×42 is two more groups of 42 than 48×42, subtracting two groups of 42, or 84, gives the final answer.

This Teacher Note appears in 5th Grade, Unit 1 Puzzles, Clusters and Towers, Investigations in Number, Data and Space K-5 math curriculum (Pearson, 2017)

It should be noted that although this strategy always works, it does not always make the problem easier to solve. It is possible to solve a multiplication problem by changing *both* numbers to make an easier problem (e.g., changing 48×42 to 50×40), but it is difficult to figure out how to adjust that answer in order to solve the original problem.

3. Creating an equivalent problem

Here are two ways students might create an equivalent problem:

$$48 \times 42 = 96 \times 21 \qquad 48 \times 42 = 16 \times 126$$

Students often call this strategy "doubling and halving" because that is how they most often create equivalent problems in multiplication. In these examples, however, you can see that the first student "doubled and halved," and the second student actually "tripled and took one-third." Using the same strategy, the students also could have changed the problem to 24×84 or 144×14.

As with changing one number to create an easier problem, it makes sense to use this strategy only if it does indeed result in a problem that is easier to solve. Sometimes a series of steps of doubling and halving can result in a much easier problem, such as

$$48 \times 42 = 24 \times 84 = 12 \times 168$$

When a student's first step is to create an equivalent problem, the next steps often include a combination of the other strategies. After a student has changed the problem to 96×21, it can be solved by breaking the numbers apart: $96 \times 21 = (96 \times 20) + (96 \times 1)$.

This strategy is an example of the associative property. One of the numbers is broken apart by multiplication and then the associative property is applied. Two examples are shown below.

$$\begin{aligned} 48 \times 42 = 48 \times (2 \times 21) &= (48 \times 2) \times 21 \\ &= 96 \times 21 \end{aligned}$$

$$\begin{aligned} 48 \times 42 = (16 \times 3) \times 42 &= 16 \times (3 \times 42) \\ &= 16 \times 126 \end{aligned}$$

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