Teacher Note

Addition Strategies

Students' strategies for addition fall into two basic categories: (1) breaking the numbers apart and then adding these parts, and (2) changing the numbers to numbers that are easier to add. In order to use these strategies, students must understand the meaning of addition and have a good mental model of what is happening in the problem. They must be able to look at the problem as a whole, think about the relationships of the numbers in the problem, and choose an approach that they can carry out easily and accurately.

By the end of this unit, third-graders should be able to break apart 3-digit numbers in a variety of ways, add the parts accurately, keep track of all the parts of a problem, and combine the parts to find the sum of the original problem. They should feel comfortable and confident with at least one strategy and should be using it efficiently—adding the largest or most reasonable parts of the number and using a minimum number of steps. They should also be familiar with strategies that involve changing the numbers to create an easier problem to solve and should be able to adjust accurately for the change.

Below are examples of students' strategies for solving the following problem:

349 + 175 =

Although the steps for each strategy are all written out in this **Teacher Note,** in practice students gradually learn to carry out many of these steps mentally, jotting down just what they need to keep track of partial sums.

Breaking the Numbers Apart

In strategies that involve breaking numbers apart and then adding the parts, students use their understanding of the ways in which numbers can be decomposed to solve the problem.

Set A, Adding by place In the solutions in Set A, students break the numbers apart by place value, add each place, and then find a final total. Students often call these approaches "adding by place" or "adding 100s, 10s, and 1s."

349 + 175 =

Philip's strategy
300 + 100 = 400
40 + 70 = 110
9 + 5 = 14
400 + 110 + 14 = 524

Chiang's strategy
300 + 100 = 400
30 + 70 = 100
10 + 100 = 110
9 + 5 = 14
400 + 110 + 14 = 524

Philip started with the largest place, adding 100s, then 10s, then 1s. Chiang did the same, but further broke 40 into 30 and 10 in order to use the known combination 70 + 30. This is an example of a step that students can learn to carry out mentally.

Students should also become familiar with vertical notation for this method:

349		349	
+ 175		+ 175	
400	*(300 + 100)*	14	*(9 + 5)*
110	*(40 + 70)*	110	*(40 + 70)*
14	*(9 + 5)*	400	*(300 + 100)*
524		524	

The expressions next to each partial sum indicate which parts of the numbers are added. Recording these expressions helps students understand the vertical notation, but students are not expected to include these expressions in their own notation.

Place-value models, such as 100 grids or stickers that come in sheets of 100, strips of 10, and single stickers, help students visualize what is happening when the numbers are broken apart by place and then added. For example, a student might describe part of the problem 349 + 175 this way:

"When I had to add 40 plus 70, I thought of it as 4 strips of (10) stickers and 7 strips. Three strips and seven strips make 10 strips, and 10 strips is a whole sheet (of 100). Then there's one strip left over—it's a hundred and ten (110)."

The U.S. regrouping algorithm, which some third-grade students may know, is also an example of adding by place. Rather than beginning with the largest place, as students often do naturally, this algorithm begins with the smallest place. It also includes a shorthand way of notating the value of the numbers as the digits in each place are added. For many third-graders, the compressed notation of this algorithm can obscure both the place value of the numbers and the meaning of each step. This can lead to a more rote approach to solving addition problems when students are solidifying their understanding of the base-ten number system and the operation of addition in Grade 3—steps in students' development of computational fluency that take time and practice.

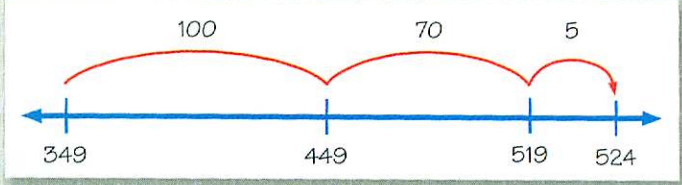
After students have developed good, efficient algorithms that they understand and can carry out easily, such as adding by place, some may also become fluent in this traditional algorithm. Others will continue to use adding by place or adding on in parts fluently, which will also serve them well for their computation needs now and as adults. The "carrying" algorithm is not addressed directly in Grade 3, although some students may be able to use it with understanding. Note that the vertical notation of adding by place shown on the whiteboard, where the ones are added first, is closely related to the steps in the standard algorithm but makes these steps more transparent. When students use the standard algorithm, demonstrate this form of notation and help students compare the two. Students who use the standard algorithm should also learn other strategies that demonstrate their flexibility with and understanding of addition. The U.S. algorithm is included in a study of strategies for addition in Grade 4.

Set B, Adding one number in parts In Set B, students break up one of the addends into parts and then add these parts, one at a time, to the other number. Students may refer to this set of solutions as "adding one number in parts."

349 + 175 =

Jane's strategy	**Adam's strategy**	**Gina's strategy**
349 + 100 = 449	349 + 100 = 449	175 + 300 = 475
449 + 70 = 519	449 + 50 = 499	475 + 25 = 500
519 + 5 = 524	499 + 25 = 524	500 + 24 = 524

Jane and Adam started with 349 but broke 175 up in different ways (100 + 70 + 5 and 100 + 50 + 25). Gina started with 175 instead and broke up 349 (300 + 25 + 24). When students use this strategy, they should be encouraged to add the largest "chunks" of numbers possible while still making sense of the problem and the numbers. Students often use a number line to represent their thinking when using this strategy.



Changing the Numbers

In this category of addition strategies, students change one or both of the numbers to what they may call "landmark" or "friendly" numbers—generally multiples of 10 or 100. These strategies require that students understand how to compensate for any changes they make. Because many third-graders are still solidifying that understanding, you may find that these strategies are not yet accessible to all students. They should nonetheless be explored and discussed when the numbers in the problem lend themselves to this approach. Addition strategies that involve changing the numbers will be explored further in Grade 4.

Set A, Changing the numbers and adjusting the sum In Set A, students change the numbers to multiples of 10 to create easier addition problems. Students often call this kind of solution "changing to a landmark."

349 + 175 =

Kelly's strategy
350 + 175 = 525
525 − 1 = 524

Elena's strategy
349 + 200 = 549
349 − 25 = 52

Denzel's strategy
350 + 200 = 550
550 − 25 − 1 = 524

After students have changed one or both numbers to a landmark and found the sum, they have to decide what to do to the sum to compensate for their initial changes. Kelly simply added 1 to 349, and then had to subtract 1 to get the final answer. Elena used a similar strategy, adding 25 to 176, then subtracting 25 at the end. Denzel changed both numbers to landmarks by adding 1 and 25, added them, then subtracted the 25 and the 1 that had been added.

Note that, as in other examples in this **Teacher Note,** students may carry out other in-between steps either mentally or in written form. For example, the first student might add 350 and 175 by adding on parts of 175 to 350: 350 + 50 + 100 + 25.

Set B, Creating an equivalent problem Sometimes students change the numbers in an addition problem to create an equivalent problem that is easier to solve.

349 + 175 =

Benjamin's strategy
324 + 200 = 524

Keisha's strategy
400 + 124 = 524

In these examples, an increase in one addend is matched by an equal decrease in the other addend so that no additional adjustment is needed after the total has been found. Benjamin subtracted 25 from 349 and added 25 to 175; Keisha added and subtracted 51.

In these solutions for this particular problem, creating an equivalent problem that is easier to solve requires adding and subtracting either 25 or 51. It is more likely that third-graders would save this method for a problem in which a very small number, typically 1 or 2, must be added and subtracted. For example, students might change 398 + 175 to the equivalent expression 400 + 173.

Creating equivalent problems that are easier to solve is addressed in this unit for the first time and will be revisited in Grade 4.