

In the course of articulating, representing, and justifying their ideas about such general claims, students in the elementary grades are beginning to engage in proving—a central part of mathematics. They consider the questions: Does this generalization apply to *all* numbers (in the domain under consideration, usually whole numbers)? Why does it work? *How do you know?* In two of the number and operations units in each grade, 2–5, you will find a Teacher Note that focuses on proof and justification. These Teacher Notes provide examples of the ways that students at that grade level engage in proving and how their proofs, based on representations, are related to the proofs a mathematician might carry out.

Examples of the general claims highlighted in the curriculum in Grades K–2 are as follows:

- Counting the same set of objects in different orders results in the same count.
- If one number is larger than another, and the same number is added to each, the first total will be larger than the second: $3 + 5 > 2 + 5$.
- You can add two numbers in either order: $6 + 3 = 3 + 6$.
- If you add an amount to one addend and subtract it from another addend, the sum remains the same: $6 + 6 = 12$; $7 + 5 = 12$.
- Addition and subtraction are related. If adding two numbers gives a certain sum, then subtracting one of the addends from the sum results in the other addend: $6 + 7 = 13$; $13 - 7 = 6$; $13 - 6 = 7$.
- You can break numbers into parts to add them: $6 + 8 = 6 + (4 + 4) = (6 + 4) + 4$.
- If you add two even numbers, the sum is even. If you add two odd numbers, the sum is even. If you add an even number and an odd number, the sum is odd.

Some of the generalizations investigated in Grades K–2 are revisited in Grades 3–5 with higher numbers and more

complex problems. In addition, new general claims are investigated. Examples of general claims highlighted in Grades 3–5 are as follows:

- If you add the same amount to both numbers in a subtraction problem, the difference does not change: $145 - 97 = 148 - 100$.
- You can multiply two numbers in either order: $32 \times 20 = 20 \times 32$.
- You can break numbers into parts to multiply them, but each part of each number must be multiplied by each part of the other number: $7 \times 24 = 7 \times (20 + 4) = (7 \times 20) + (7 \times 4)$.
- Multiplication and division are related. If multiplying two numbers gives a certain product, then dividing that product by one of the original factors results in the other factor: $9 \times 8 = 72$; $72 \div 8 = 9$; $72 \div 9 = 8$.
- A factor of a number is a factor of multiples of that number: 3 is a factor of 15; 15 is a factor of 30, so 3 is a factor of 30.
- If you double (or triple) one of the factors in a multiplication problem and halve (or third) the other, the product remains the same: $164 \times 4 = 328 \times 2$.